Statistical Hadronization phenomenology

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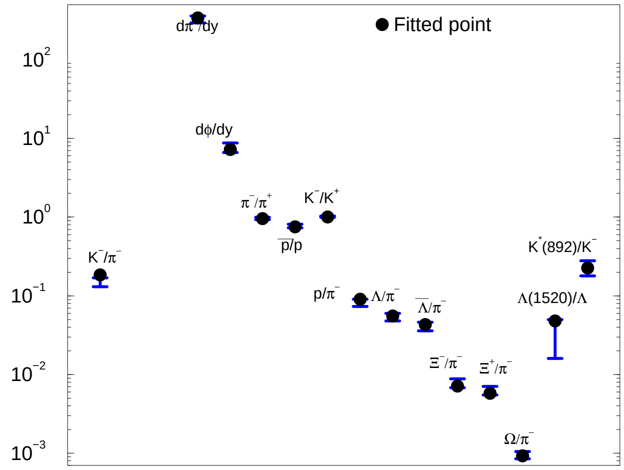
http://www.physics.arizona.edu/∼torrieri/SHARE/share.html

Statistical models: considerable phenomenological success



Plots like this shown at most workshops on the subject.

At Au-Au and p-p RHIC collisions, fitting T, μ_B,other parameters $\Rightarrow$ a "nice-looking" plot with nearly all particles accounted for. But does this prove "equilibrium" is really there?

- We always knew soft hadronic abundances were approximately exponential. Are $T, \mu, Volume$ "real", or are they "epicycles"?

- Becattini has done thermal fits for $p-p, e^+ - e^-$. Does that mean these systems are equilibrated? Or not? most points fit, some fail quite badly. but, some particle yields fail in A-A systems as well. When does true equilibration kick in?

First question: Can we test statistical hadronization?
Fluctuations: Statistical mechanics falsifier

Statistical mechanics (in fact, all statistics)predicts a relationship between "averages" ($\langle X \rangle$) and fluctuations ($\langle (\Delta X)^2 \rangle$.
The validity of statistical mechanics is founded on fluctuations going to 0 in certain limits.

A good check for the consistency of the statistical model is fitting both yields and fluctuations with same parameters! And it has never been done until now!

The statistical model:

$$N = \int \mathcal{M} \prod_i \frac{d^3\vec{p}_i}{E_i} \delta_E \delta_Q$$

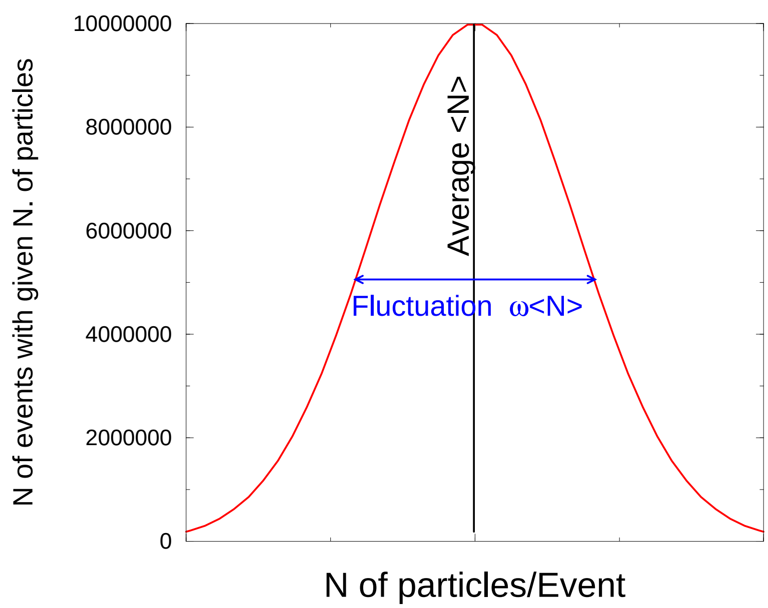
$\mathcal{M} \rightarrow constant$ (dynamics $\rightarrow$ phase space)

$$P_N = \frac{\Omega_N}{\sum_n \Omega_n} \qquad \Omega = \int \prod_i \frac{d^3\vec{p}_i}{E_i} \delta_E \delta_Q$$

Observables:

$$< N > \quad , \quad \omega = \frac{(\langle \Delta N \rangle^2 \rangle}{\langle N \rangle}, \quad \text{higher comulants}$$

calculable through partition function



Several ways of defining $\delta_{E,Q} \rightarrow$ Ensembles.

Ensembles , or how to deal with conservation laws
$\lim_{V\to\infty}^{N/V=const} < N >$ same in $\forall$ ensembles. not ω

Micro-canonical : EbyE conservation

$$\delta_E \delta_Q = \delta\left(\sum_i E_i - E_T\right)\delta\left(\sum_i Q_i - Q_T\right) \quad \omega_E = \omega_Q = 0$$

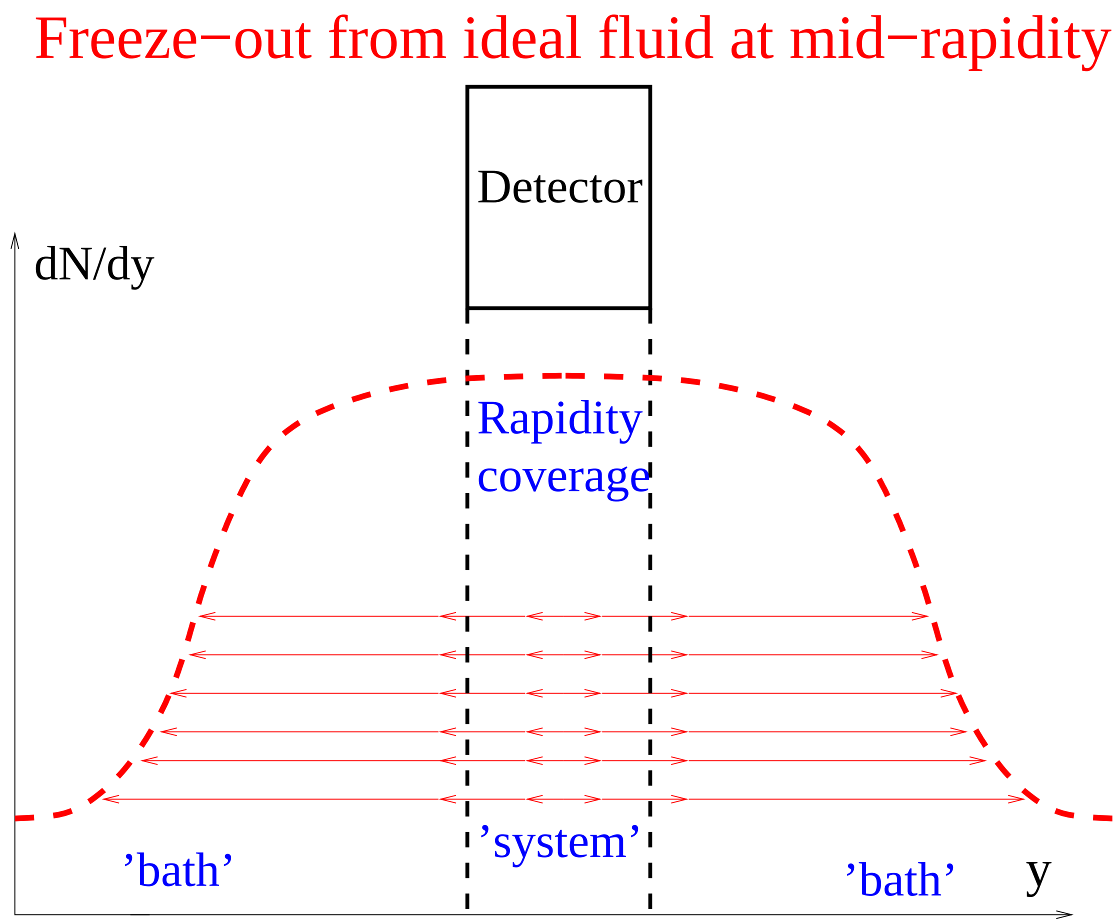
Canonical : Energy conserved on average
Appropriate for system in equilibrium with bath

$$\delta_E \to \delta\left(E_T - < E >\right) \qquad \omega_E \sim 1$$

Grand Canonical : Charge conserved on average

$$\delta_Q \to \delta\left(Q_T - < Q >\right) \qquad \omega_E \sim \omega_Q \sim 1$$

Appropriate for detector sampling part of a fluid



Boost invariance: Rapidity ⇔configuration space

- Mid-rapidity ⇔system
- Peripheral regions ⇔bath

⇒ Grand Canonical ensemble needs to be used!
NB: This is an experimentally verifiable statement: The dependance of fluctuations on yields is Ensemble-specific (Begun,Gorenstein,Gazdzicki,Zozulya),so an incorrect ensembe will fail to describe both

Cleymans, Redlich, PRC 60, 054908 (1999):

$$\left[\frac{dN}{dy}\right]_{b.i.} \sim < N >_{4\pi} \quad \left[\frac{d(\Delta N)^2}{dy}\right]_{b.i.} \sim (\Delta N)^2_{4\pi}$$

- All details of flow and freeze-out integrate out

- Up to Normalization, $< N >, \omega$ calculable from Grand Canonical T, λ_i

$$\left. \begin{array}{l} Ideal \quad hydro \\ Freezeout@const. \quad T \end{array} \right\} \begin{array}{l} Statistical \quad model \quad \underline{fits \quad well} \\ < N > \quad \underline{AND} \quad \omega_N \end{array}$$

So lets see how the statistical model does!
But which one?

Grand canonical statistical hadronization

All particles described in terms of T and $\lambda_{q,s,I3}$.
Detailed balance: $\lambda_{\overline{q}} = \lambda_q^{-1}$Integral can be done in rest-frame wrt flow using Bessel function K_2

$$\langle N_i \rangle = \lambda_i \frac{\partial \ln Z}{\partial \lambda_i} = V' \sum_{n=1}^{\infty} (\mp 1)^{n+1} \frac{\lambda_i^n}{n} F(m, nT)$$

$$\left\langle (\Delta N_i^2) \right\rangle = \lambda_i^2 \frac{\partial^2 \ln Z}{\partial \lambda_i^2} = V' \sum_{n=1}^{\infty} (\mp 1)^{n+1} \frac{\lambda_i^n}{n} C_n^{2+n-1} F(m, nT)$$

$$F(m, T) = m_i^2 T K_2 \left(\frac{m_i}{T} \right)$$

Resonance feed-down

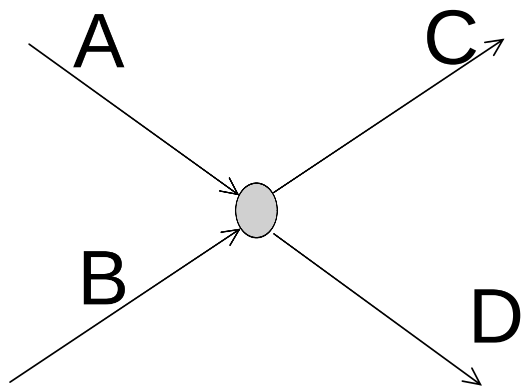
$$\langle N_i \rangle = \langle N_i \rangle^{direct} + \sum_j b_{j \to i} \langle N_j \rangle$$

$$\Delta N_i^2 = \Delta N_i^2 + \sum_j \left[\underbrace{b_{j \to i} (1 - b_{j \to i}) N_j}_{Fluctuation \quad of \quad j \to i} + \underbrace{b_{j \to i}^2 \left\langle (\Delta N_j)^2 \right\rangle}_{Fluctuation \quad of \quad N_j} \right]$$

Fluctuations of quantities like $Q = N_+ - N_-$ or N_1/N_2 also contain correlations due $j \to N_1 N_2$.

Lots more on this later

Chemical Equilibrium Detailed balance:



$$\lambda_A \lambda_B = \lambda_C \lambda_D \longrightarrow \lambda_A = \lambda_{\bar{A}}^{-1}$$

So Chemical potentials for conserved quantities

$$\lambda_i = \lambda_u^{u-\overline{u}} \lambda_d^{u-\overline{u}} \lambda_s^{s-\overline{s}}$$

Fit $T, \lambda_{q,s,I3}$ to yields and ratios $\rightarrow T \sim 165 MeV$ at upper energy SPS and RHIC

Non-Equilibrium

- A dynamically expanding system might well not be in detailed balance, especially if phase transitions are involved
- Parametrize deviation from equilibrium by γ_i

$$\lambda_i \rightarrow \lambda_i^{\mathrm{eq}} \gamma_u^{u+\overline{u}} \gamma_d^{u+\overline{u}} \gamma_s^{s+\overline{s}} \qquad \gamma^{eq} = 1$$

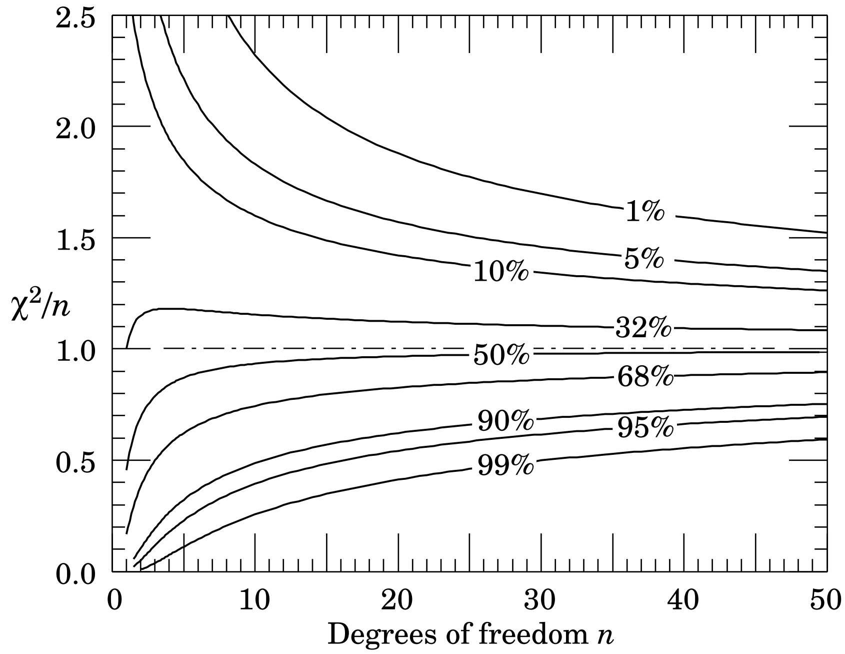
- Csorgo and Csernai, '94: Supercooling might necessary to conserve entropy
- Rafelski, Letessier, '99: Freeze-out from Entropy-rich QGP $\rightarrow T = 140, \gamma_q = 1.6$
- Greater strangeness at equilibrium QGP than equilibrium HG $\Rightarrow$ Hadron gas $\gamma_s > 1$

When γ_q, γ_s put in as fit parameters, T drops to 140 MeV, γ_q rises to ~ 1.6 and γ_s to ~ 2 at SPS and RHIC. discovery of super-cooled phase transition or over-fitting?!

Third and fourth questions

2 statistical models on the market!

Equilibrium statistical model Braun-Munziger,Redlich,...	Non-equilibrium Rafelski, Letessier, GT
oven-like	Explosion-like
High T (~ 165 MeV)	Supercooled (~ 140MeV)
Equilibrium ($\gamma_{q,s} = 1$)	Over-saturation ($\gamma_{q,s} > 1$)
Staged freeze-out	Sudden freeze-out
Resonances don't freeze-out at same T	Resonances freeze-out at same T
Strangeness systematics due to approach to thermodynamic limit (Canonical $\rightarrow$ GC)	Strangeness systematics due to phase transition γ_s/γ_q grows since more s/Q in QGP
No info on phase transition	First order or sharp cross-over
No info on early phase	Early phase probed



- Statistical significance, the probability of getting χ^2 with n DoF given that "your model is true", is a quantitative measure of your fit's goodness

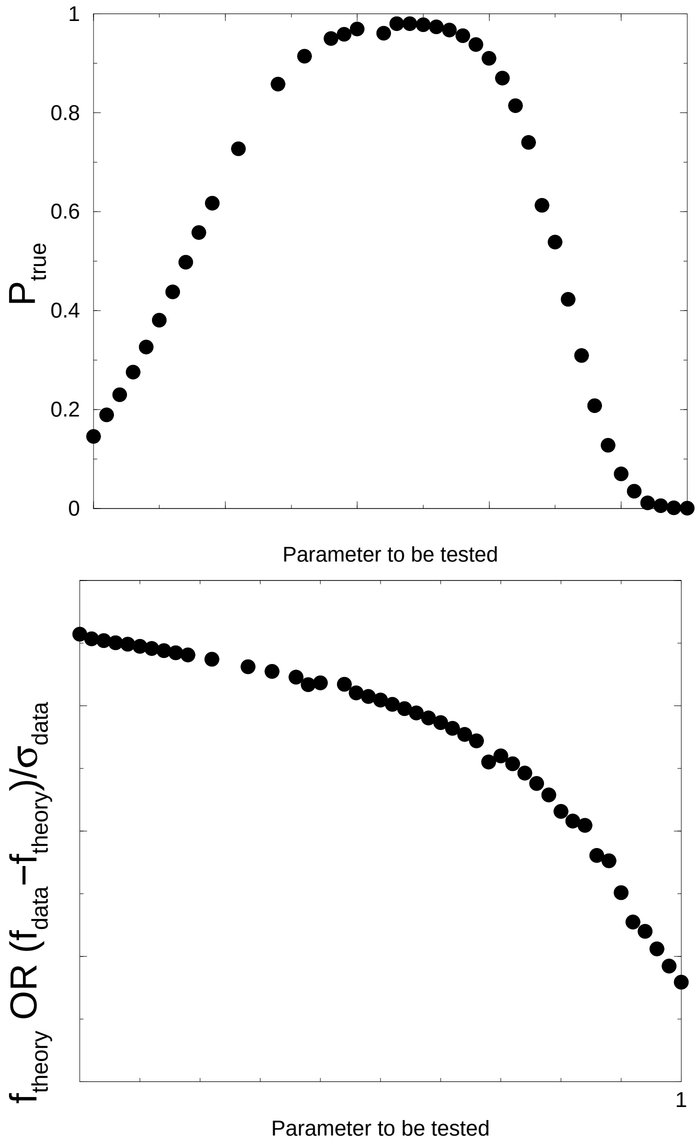
- models with different N_{dof} can be compared

- With few DoF, "nice" looking graphs can have a very small statistical significance.

- It is said that you can fit an elephant with enough parameters. Maybe so, but if you are honest, you won't get a good statistical significance.

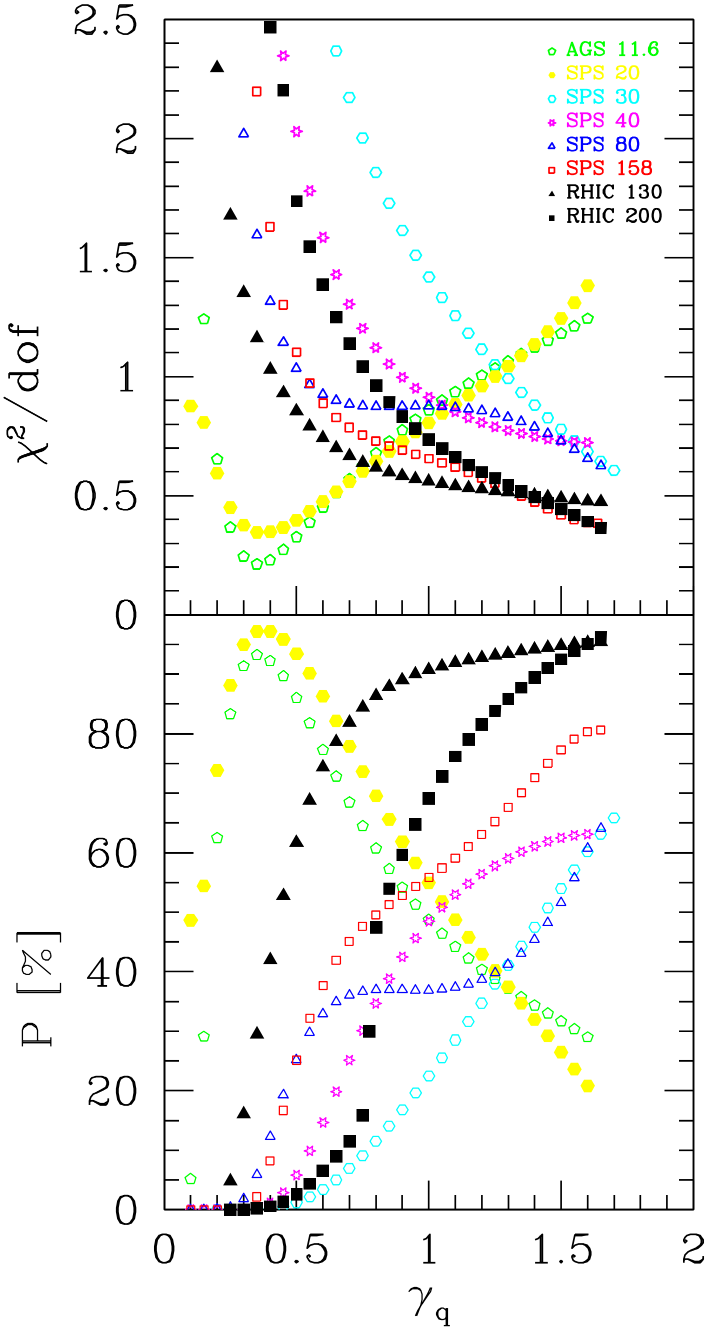
Non-trivial correlations/data-point sensitivity can be analyzed by Profiles in statistical significance

All other parameters at their best fit value for point in abscissa



Let's apply this to γ_q!

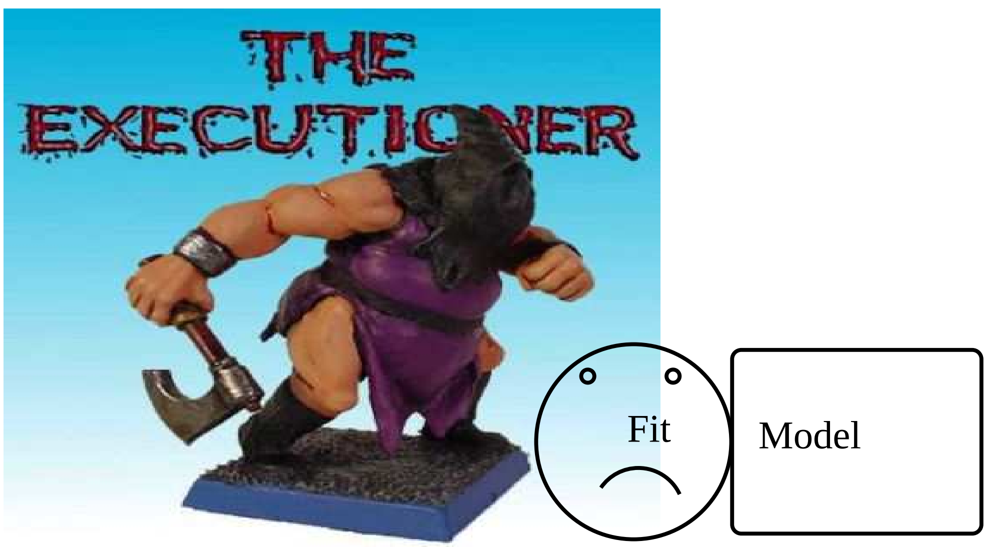
(Letessier and Rafelski, nucl-th/0504028)



- Maximum for SPS and RHIC is at $\gamma_q > 1$, suggesting this is probably not over-fitting
  - $\left(\frac{\gamma_s}{\gamma_q}\right)_{\gamma_q>1} > \left(\frac{\gamma_s}{\gamma_q}\right)_{\gamma_q=1} \Rightarrow$ More $\frac{\Lambda}{p}, \frac{\Xi}{\Lambda}, \frac{\Omega}{\Xi}$
  - Lower T$\Rightarrow$ less resonances agrees with Experiment

- But equilibrium not ruled out!.
  T and γ_q strongly correlated, making their individual determination difficoult

We need this guy:

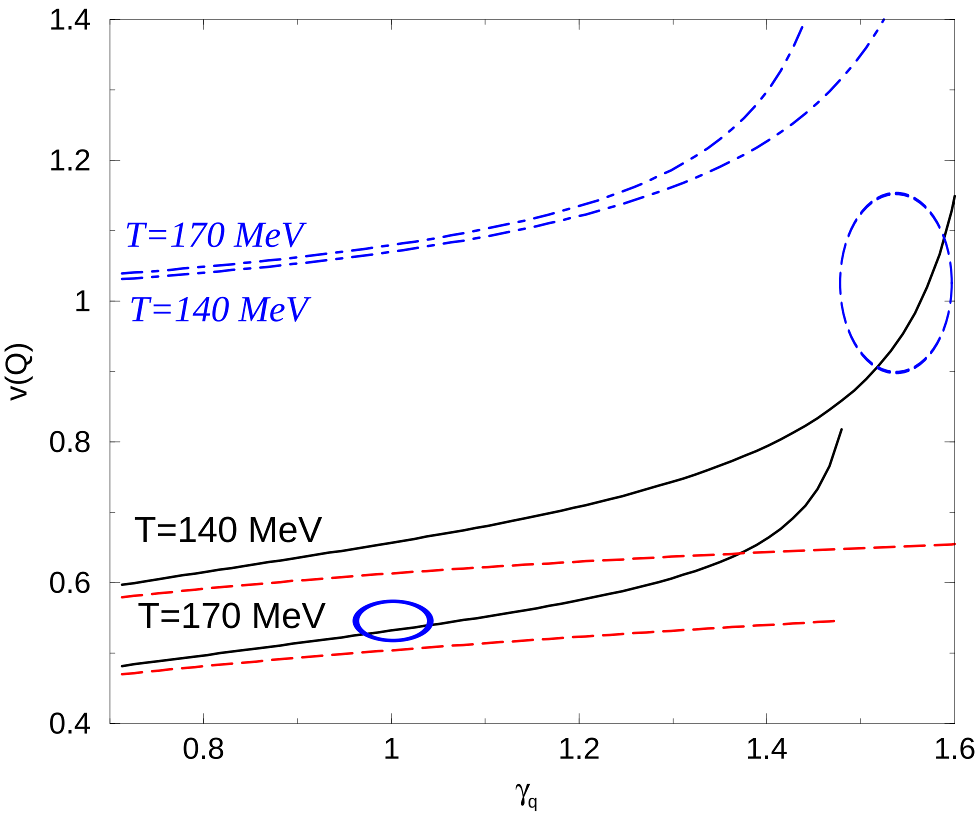


ie, further data...

- That one EXPECTS statistical models to describe

- That is capable of determining γ_q,T, post-emission reinteraction.

Yields and Fluctuations: Non-equilibrium



T increase $\Rightarrow$ π Fluctuations decrease because of enhanced resonance production
Resonances affect correlations

over-saturation ($\gamma_q > 1$) $\Rightarrow \pi$ Fluctuations increase faster than yields because of BE corrections

$$\gamma_q^2 e^{m_\pi/T} = 1-\epsilon \Rightarrow \frac{\langle N_\pi \rangle}{V} \sim \epsilon \qquad \frac{\langle (\Delta N_\pi)^2 \rangle}{V} \sim \epsilon^2$$

$\gamma_q > 1$ affects fluctuations

A small problem: Volume fluctuations are not well understood, and show up in all $< N^2 > - < N >^2$. Avoid them choosing observables such as

- $(\Delta Q)^2$. $\frac{\langle Q \rangle}{V}$small, so is $\Delta V \frac{\langle Q \rangle}{V}$ (Jeon, Koch)

- fit $\left\langle (\Delta V)^2 \right\rangle$

- undestand $\left\langle (\Delta V)^2 \right\rangle$ (KNO scaling:$(\Delta V)^2 \sim < V >$, pressure ensemble!)

- Fluctuations of ratios(Jeon, Koch), Volume fluctuations irrelevant!

$$\sigma^2_{N_1/N_2} = \frac{\left\langle (\Delta N_1)^2 \right\rangle}{\langle N_1 \rangle^2} + \frac{\left\langle (\Delta N_2)^2 \right\rangle}{\langle N_2 \rangle^2} - 2 \underbrace{\frac{\langle \Delta N_1 \Delta N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle}}_{Resonance \quad correlation}$$

Points to note

- Fluctuations of ratios have a resonance-derived correlation term!
  Correlation appears at chemical freeze-out, is not destroyed by rescattering
  (Undetectable resonances still correlate!)

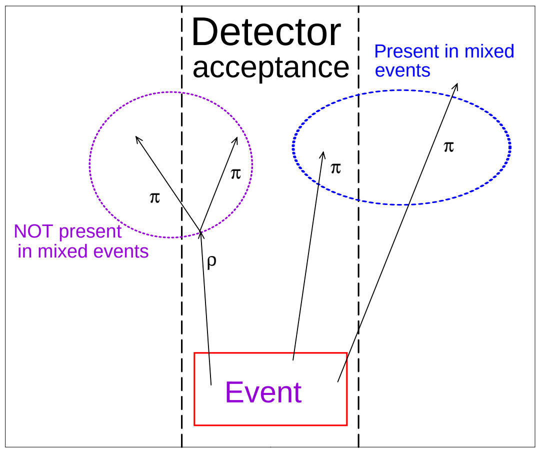
- Fluctuations of ratios depend on volume!

$$\sigma^2_{N_1/N_2} \sim \frac{1}{\langle V \rangle T^3}$$

  Hence, a fit with fluctuations of ratios needs a "normalization" fit parameter. But fitting ratios and multiplicities $\sim \langle V \rangle T^3$ constrains normalization (along with T and γ_q) tightly.

A big problem: Experimental acceptance

All measurements depend on rapidity, p_T cuts etc. of detector. For fluctuations, these can dominate



Pruneau, Gavin, Voloshin: use dynamical fluctuations

$$\sigma_{dyn} = \underbrace{\sigma}_{Physics+Detector\ \ effects} - \underbrace{\sigma_{stat}}_{Detector\ \ effects}$$

$\sigma_{stat} \sim \frac{1}{<N_1>} + \frac{1}{<N_2>}$ obtained via mixed events

Any phase space cuts should produce same fluctuation in mixed event sample, so σ_{dyn} robust against detector acceptance but needs more parameters ("volume") to be described. Can use it in fit, including yields at same centrality as σ_{dyn} .

Resonances+acceptance is still a problem!

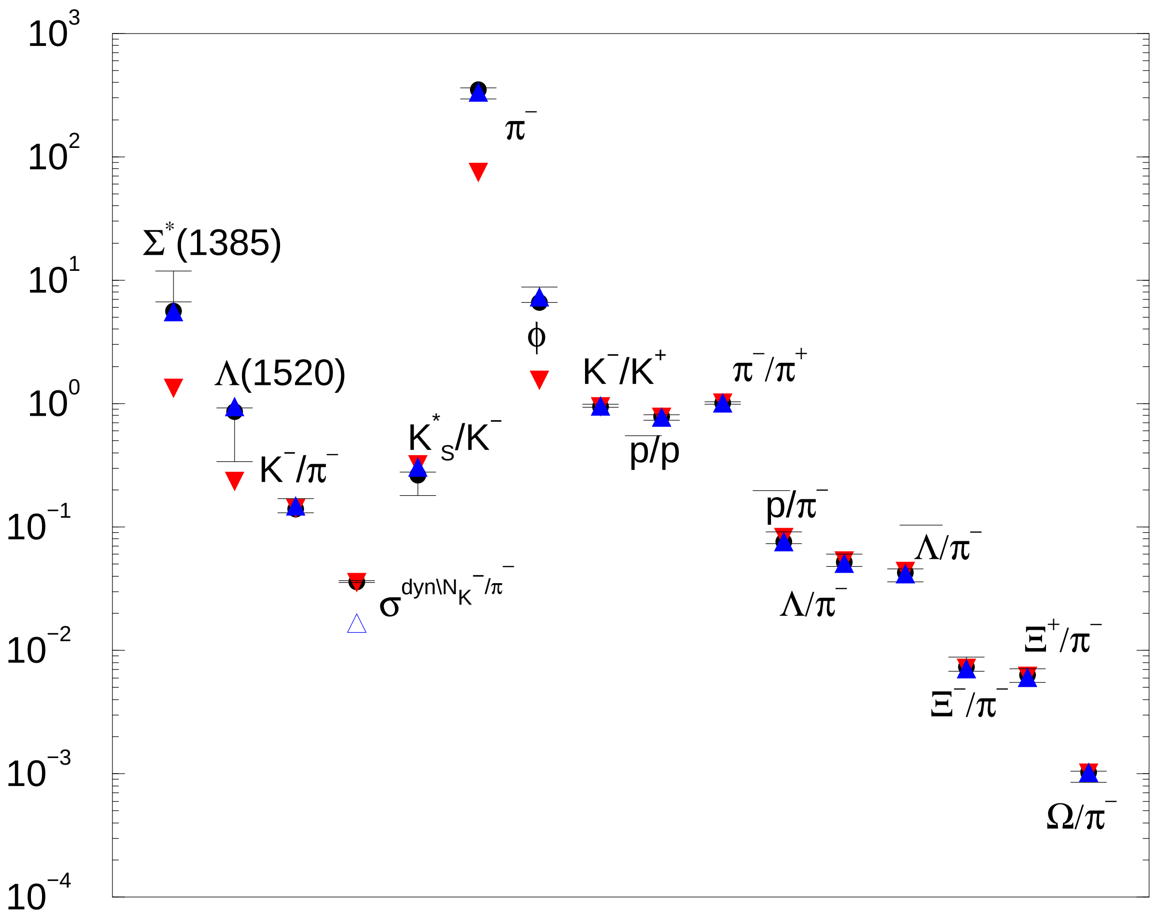
Current RHIC data (K^-/π^- and K^+/π^+ fluctuations) does not have this problem, but future K^+/π^- etc. will

Fits at 200 GeV

- $\sigma^{dyn}_{K/\pi}$: Supriya Das et al [STAR] nucl-ex/0503023
  - No common resonances $\rightarrow$ no need to worry about correlation corrections
  - Common resonances would be nice, through! (see predictions)

- Ratios:O. Barannikova et al [STAR] nucl-ex/0403014

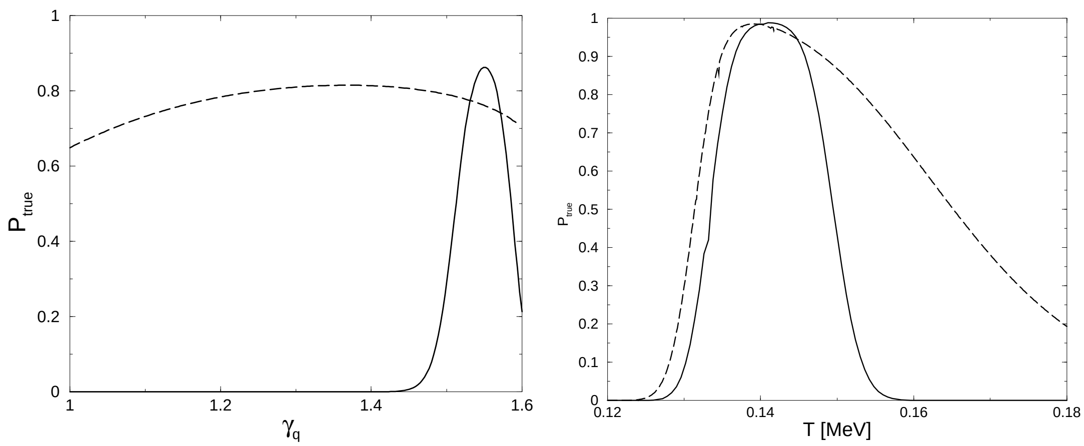
NB: All preliminary



- Equilibrium fit yields only → Underestimates σ^{dyn} by many standard deviations

- Equilibrium fit with fluctuations → Too small $\langle V \rangle$ to describe multiplicities

- $\gamma_q > 1$ → acceptable description of both yields and fluctuations



- Fluctuations do indeed fix tightly γ_q at above 1

- Best fit T at ~ 140 MeV, describes $K^*, \Lambda(1520)$, Σ^* (to 1.5 s.d.)

- All data preliminary! But approach promising!

How much reinteraction between T_{chem} and T_{th}?

A little/none	A lot
Who wants it?	
Non-equilibrium	Equilibrium
Spectra?	
If one includes Resonances most hadrons fit $T_{th} = T_{ch}$ (Florkowski,Broniowski GT,Rafelski,Letessier)	If no resonances $T_{\Xi,\Omega} > T_{th} \sim 100$ MeV , (STAR,PHENIX) (But resonances there!)
HBT?	
-Rapid decoupling -Fits hydro with $T_{th} = T_{ch}$(Heinz,Kolb)	Hydro+uRQMD fails! Ideas around, but no solution
Resonances?	
Hadronic $\rho, \Sigma^*, \Delta, K^*, \Lambda^*, \phi$ ($\Gamma^{-1} = 1 - 100$ fm) Found: -no evidence of of p-p/A-A m, Γ modification -Abundances compatible or exceeding $\gamma_q > 1$ fit $\forall \Gamma$ (Certainly above T=100 MeV) (STAR/GT,Rafelski,Letessier)	$\rho \to \mu^+ \mu^-$ Found broadening (Reinteraction?) (NA60/Rapp,Wambach)

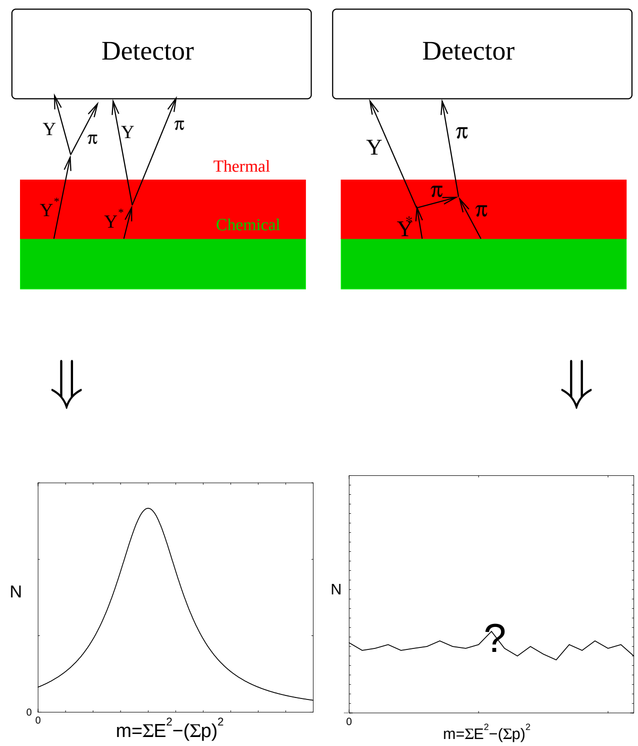
I don't fully understand this, and neither do you!

The illusion of knowledge is worse than ignorance R.Feynmann

First answer: Resonances

$\frac{K^*}{K}, \frac{\Lambda(1520)}{\Lambda}, \frac{\Xi(1530)}{\Xi}, ...$Sensitive T probe

also susceptible to in-medium re-interactions



Issues to consider:

- Any re-interaction can usually only <u>suppress</u> resonances
  - A few $\rightarrow$ rescattering>regeneration $\rightarrow$ suppression
  - A lot $\rightarrow$ <u>re-equilibration</u> at <u>lower T</u> $\rightarrow$ suppression

But some resonances ρ, Δ, Σ^* appear enhanced w.r.t. 170 MeV , never mind 100 MeV.

- In general, rescattering will depend on Γ (dimensional analysis+optical theorem)

$$N_i\left(\frac{m_i}{T},\lambda\right) \longrightarrow F\left[N_i\left(\frac{m_i}{T_{chem}},\lambda_{chem}\right),\Gamma_i \tau^{resc}\right]$$

2 ratios, such as $\frac{\Lambda(1520)}{\Lambda}$ vs $\frac{K^*}{K}$ $\Leftrightarrow$ T_{chem} and τ_{resc}

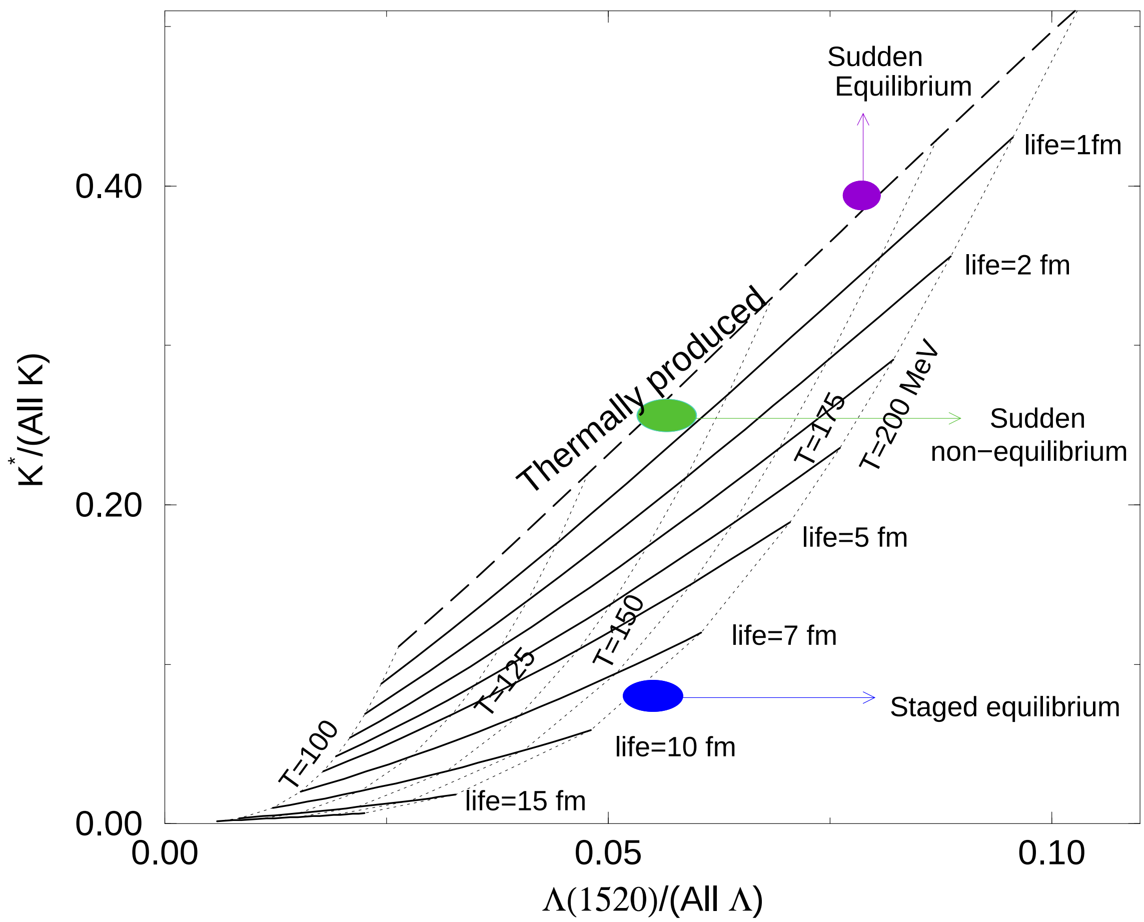
Rescattering model, GT and Rafelski, PLB, 509 239

$$\frac{dN^*}{dt} = -\Gamma N^*$$

$$\frac{d(N\pi)}{dt} = \Gamma N^* + (N\pi) < \sigma\gamma v > \frac{N_0}{V_0}\left(\frac{R_0}{R_0+vt}\right)^3$$

- Observable $(N\pi)$ pairs created through decay and destroyed through rescattering

- Density $\frac{N_0}{V_0}$ fixed by statistical hadronization, R_0 by particle multiplicity, flow from spectral fits



- People doubt this since we neglected regeneration

- Semi classical approaches such as uRQMD drastically over-estimate n. of regenerated detectable particles by mass-shell assumption

But these are just words (and models!). We still have an ambiguity. Is there a experimental way to rule out either a fast freeze-out or a long reinteracting phase? Yes! Fluctuations

Fluctuations CORRELATED by resonance decays

$$(\Delta Q)^2 = \left\langle(\Delta N)^2\right\rangle + \left\langle \Delta\overline{N})^2\right\rangle - 2\underbrace{\left(\left\langle N\overline{N}\right\rangle - \langle N\rangle\left\langle\overline{N}\right\rangle\right)}_{\rho\to N\overline{N}}$$

$$\sigma_{K/\pi} = \frac{\left\langle(\Delta K)^2\right\rangle}{\langle K\rangle^2} + \frac{\left\langle(\Delta\pi)^2\right\rangle}{\langle\pi\rangle^2} - \frac{2}{\langle K\rangle\langle\pi\rangle}\underbrace{\langle\Delta K\Delta\pi\rangle}_{K^*\to K\pi}$$

Correlation, by definition, happens at chemical freeze-out, where multiplicities are fixed! As shown in the second part of the talk, subsequent reinteraction should not change correlation.

(Up to Fluctuation from detailed balance of reactions like $Y^+\pi^+ \Leftrightarrow Y^0\pi^0$, but $\sim \left\langle(\Delta C[f])^2\right\rangle$, where $C[f]$ is Boltzmann collision term , so higher order effect)

As we know from before, however, resonance detection detects resonance abundance at thermal freeze-out!

Yields and fluctuations: Reinteraction (or not)
Consider $Y^* \to Y\pi$

$\sigma_{Y/\pi}$ probes correlation of Y and π from Y^* at chemical freeze-out.
(further rescattering/regeneration does not change the correlation.

Y^*/Y **yield** probes Y^*at thermal freeze-out (after all rescattering.

So...

- If can fit stable particles and resonances and fluctuations in same fit $\to$ no reinteraction
- If Stable particles+ Fluctuations fit gives wrong value for resonances $\to$ magnitude of reinteraction

Up until now 200 GeV data has $\sigma^{dyn}_{K^-/\pi^-}, \sigma^{dyn}_{K^+/\pi^+}$ (no resonances)

The next step: K^-/π^+ fluctuations

At RHIC this is simple, since $K^+ \simeq K^-$, $\pi^+ \simeq \pi^-$

$$\langle \pi^- \rangle \left(\underbrace{\sigma_{dyn}^{K^-/\pi^-}}_{no\ \ resonances} - \underbrace{\sigma_{dyn}^{K^+/\pi^-}}_{K^*(892)\to K^+\pi^-} \right) \simeq \frac{\langle \Delta\pi^+ \Delta K^- \rangle}{\langle K^- \rangle} \sim$$

$$\sim \left[\frac{K^*(892)}{K^-} \right]_{chemical\ \ f.o.} \qquad vs \qquad \left[\frac{K^*(892)}{K^-} \right]_{thermal\ \ f.o.}$$

From best fit (non-equilibrium) at $\Delta Y = 0.1$, $\sigma_{K^+/\pi^-} \simeq 3.10\%$ (vs $\sigma_{K^+/\pi^+} \simeq 3.61\%$ and $K^{*0}(892)/K^- \sim 0.3$.)

If that fits Evidence for sudden freeze-out!

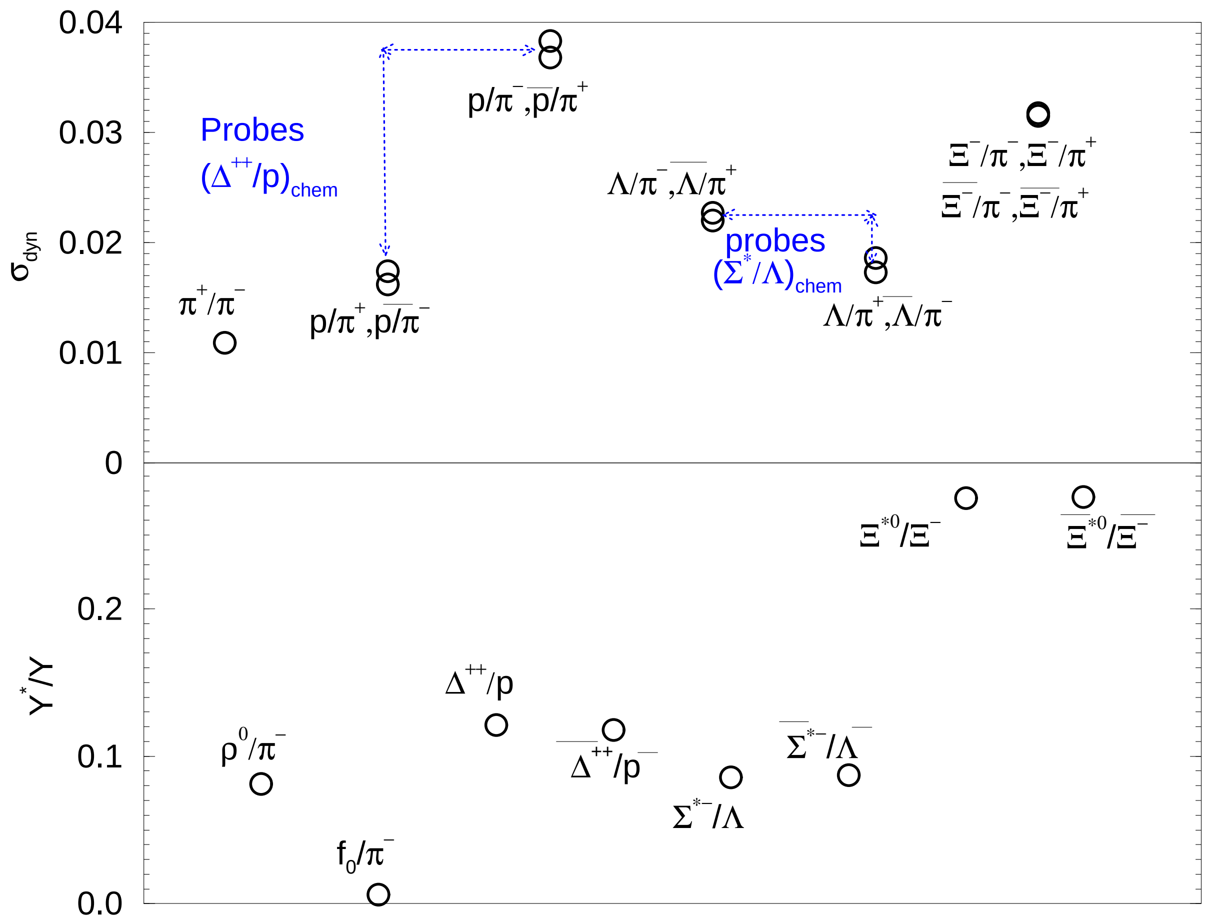
If that does not fit

- $\left[\sigma_{dyn}^{K^+/\pi^-} \right]_{exp} < \left[\sigma_{dyn}^{K^+/\pi^-} \right]_{theory}$
  $\Rightarrow$ Evidence for long re-interacting phase
- $\left[\sigma_{dyn}^{K^+/\pi^-} \right]_{exp} > \left[\sigma_{dyn}^{K^+/\pi^-} \right]_{theory}$
  $\Rightarrow$ Evidence for long re-interacting phase+K^* Melting

At SPS more complicated because of large chemical potential, but SHARE can fit!

Sudden freeze-out Predictions: $\frac{Y^* \to Y \pi}{Y} vs \sigma_{Y/\pi}$

Probe of statistical formation and post-freeze-out interactions!

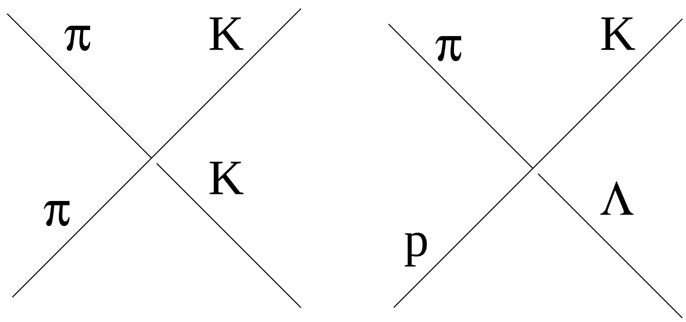


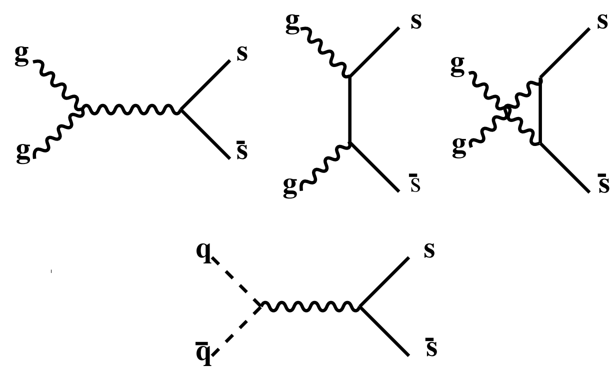
If significant discrepancies

- NO sudden freeze-out
- Difference sensitive to $T_{chem} - T_{therm}, V_{chem} - V_{therm}$

Strangeness: a probe for QGP?

Koch, Rafelski, Muller 1982, 1986: QGP kinetics more efficient at producing $s\bar{s}$ than HG kinetics



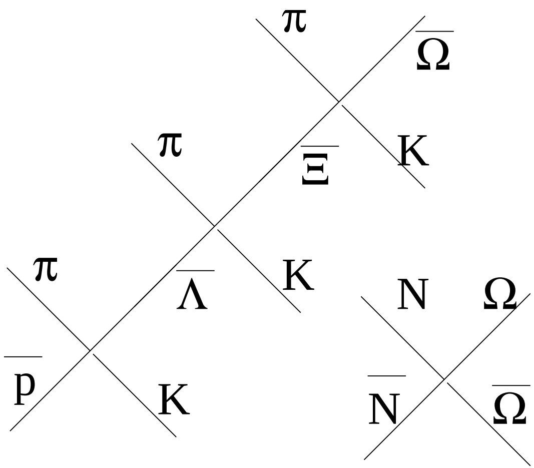


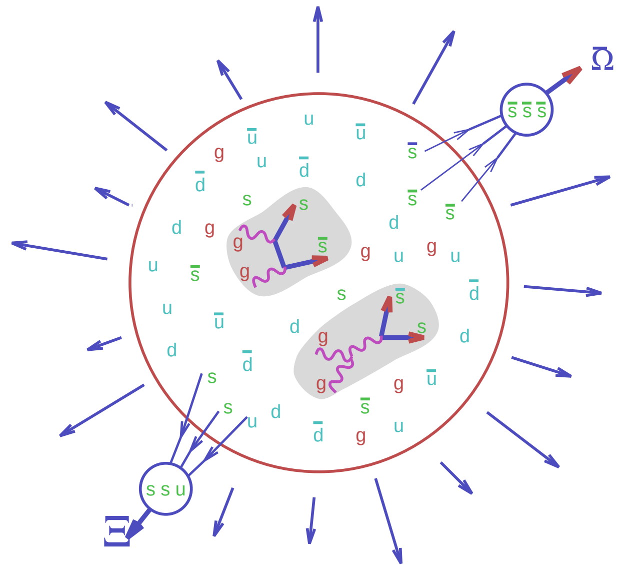
- Faster equilibration time

$$Q_{hadrons} \sim 500MeV \qquad Q_{QGP} = 2m_s \sim 200MeV$$

- More $s\bar{s}$ at equilibrium ($\gamma_s > 1$ in HG phase?)

$$\frac{m_{K,\Lambda,...}}{T} \ll \frac{m_s}{T}$$





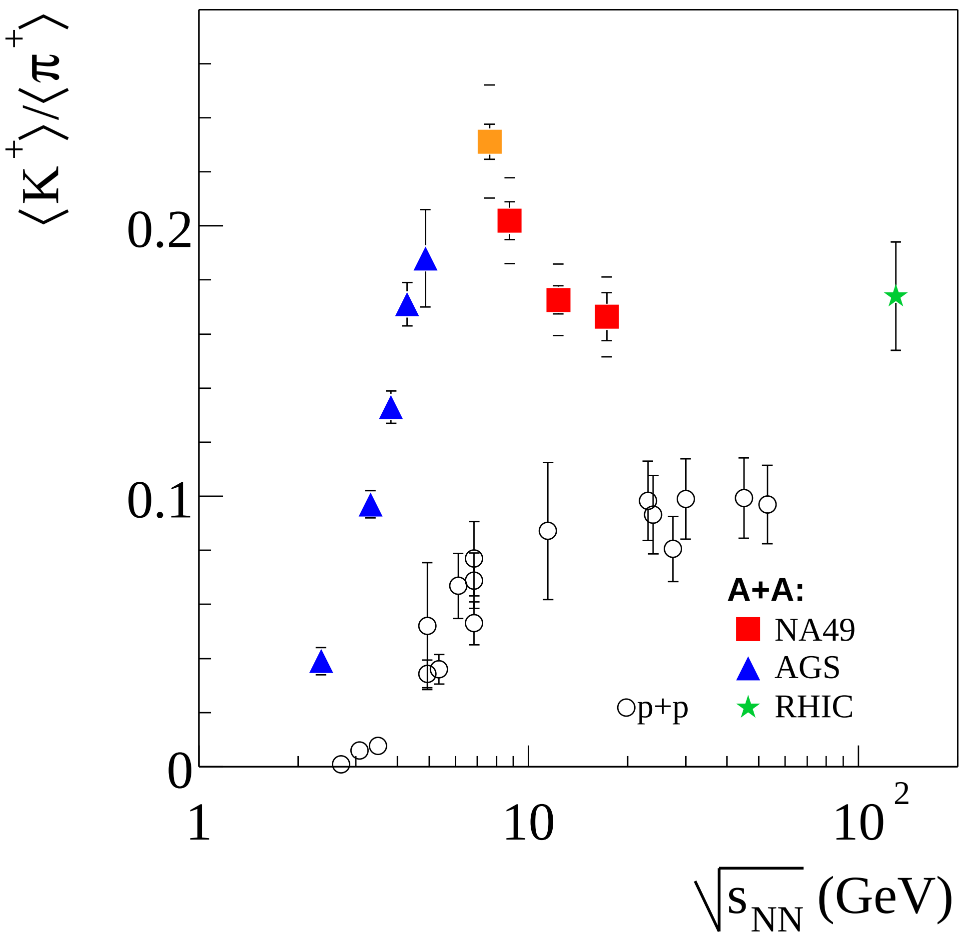
strange quark coalescence enhances multistrange ANTIbaryons with respect to hadronic production

$$\frac{3m_s}{T} >>>>> \frac{m_\Omega}{T}$$

$$Q_{N\overline{N}\to\Omega\overline{\Omega}} <<<< 3m_s$$

$$\tau_{p\pi\to\Lambda\pi\to\Xi\pi\to\Omega} <<<< \tau_s^{QGP}$$

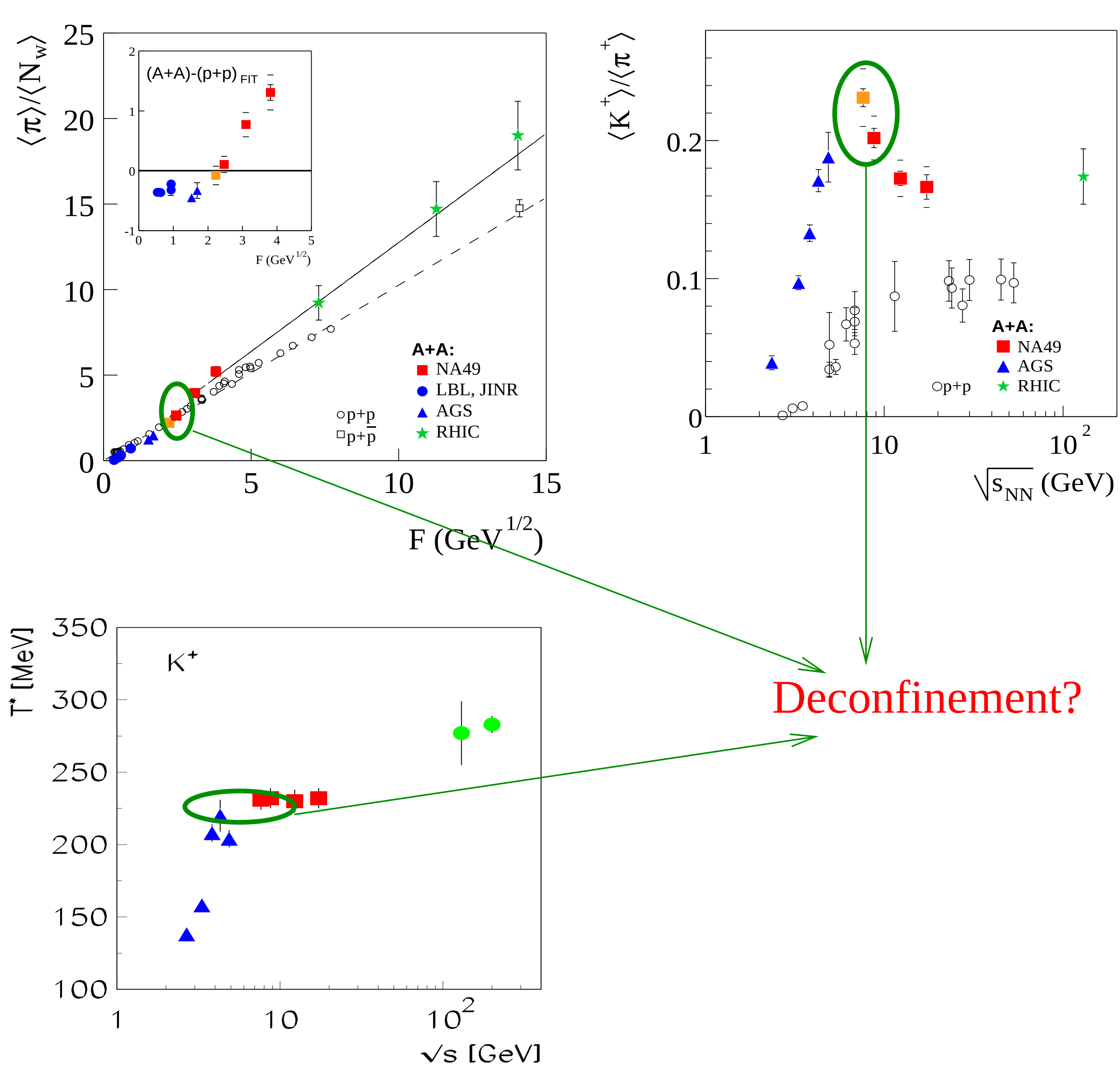
Experiment I: The "horn"



A discontinuity is observed @ $\sqrt{s}/A \sim 8GeV$ when plotting $\frac{s}{\pi}\left(\frac{K^-}{\pi^-}, \frac{\Lambda}{\pi}, ...\right)$ ratios with $\sqrt{s}$.

Nothing similar in p-p collisions

The energy of this discontinuity coincides with a shift in the energy dependence of pion yield ("the kink") and a plateau in slopes ("the plateau")



Are we seeing deconfinement?

We don't know... Looks interesting but many interpretations have been offered

- Original suggestion: Strangeness/entropy change in phase transition (Gazdzicki/Gorenstein. Kink would be evidence of enthropy density increase, step of latent heat)

- Along similar lines: Chemical non-equilibrium from phase transition (Rafelski/Letessier)
  Large entropy/strangeness content $\rightarrow \gamma_{q,s} > 1$ at deconfinement thhreshold

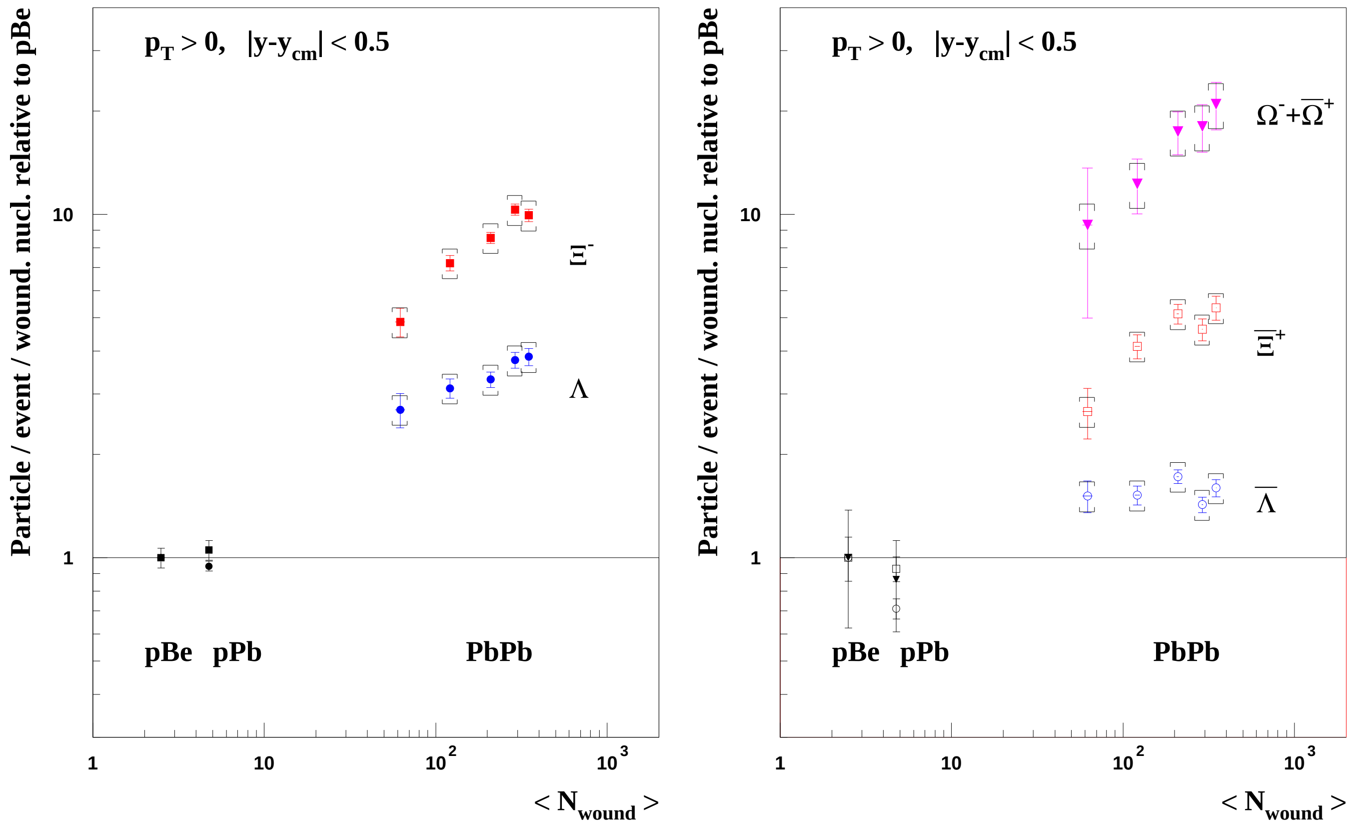
- Transition from Canonical to Grand-Canonical limits (Cleymans/Redlich)

- Transition from "Baryon-dominated" to "Meson dominated" freeze-out (Cleymans,Redlich,Kampfer,Wheaton

- $K-\pi$ non-equilibrium plus shorter interaction time at high-energy (Tomasik)

It would be great to rule out some of these!

Experiment II: Enhancement, defined as

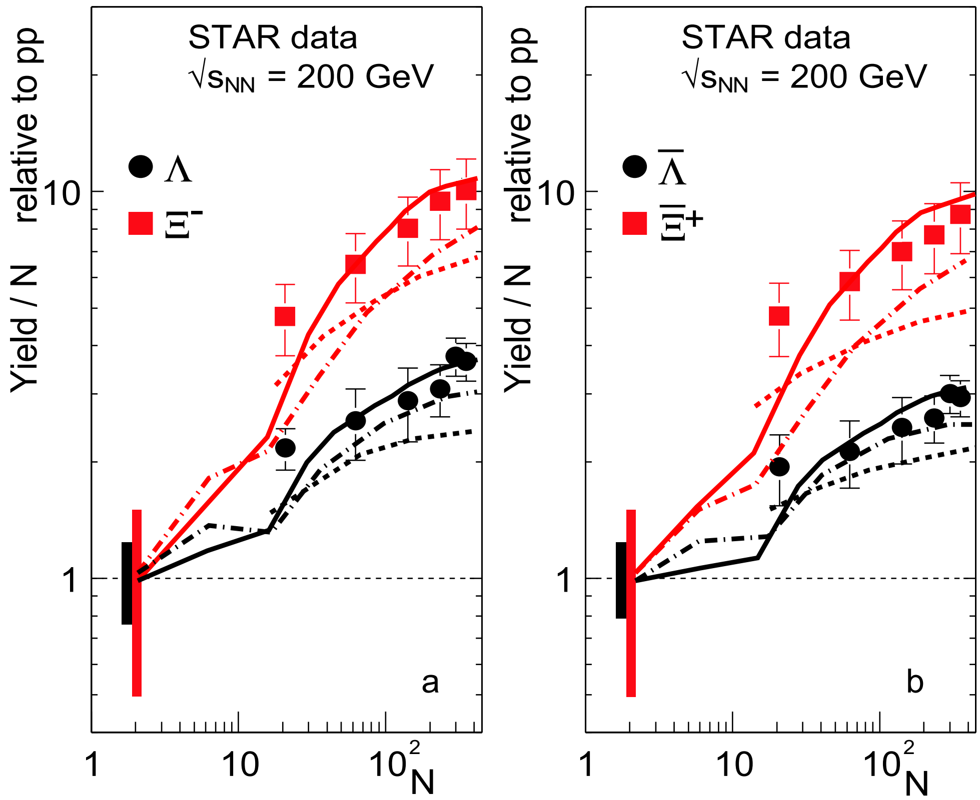


$$\frac{N^{AA}/N^{AA}_{part}}{N^{pp}/N^{pp}_{part}}$$

is definitely there, as much as ~ 20 for $\overline{\Omega}$. But the interpretation of this has been subject to controversy

When fitting yields a consistent picture emerges

Extra strangeness is due to higher $\gamma_s > 1$and Volume, as expected if A-A system lived in phase efficient at producing strangeness

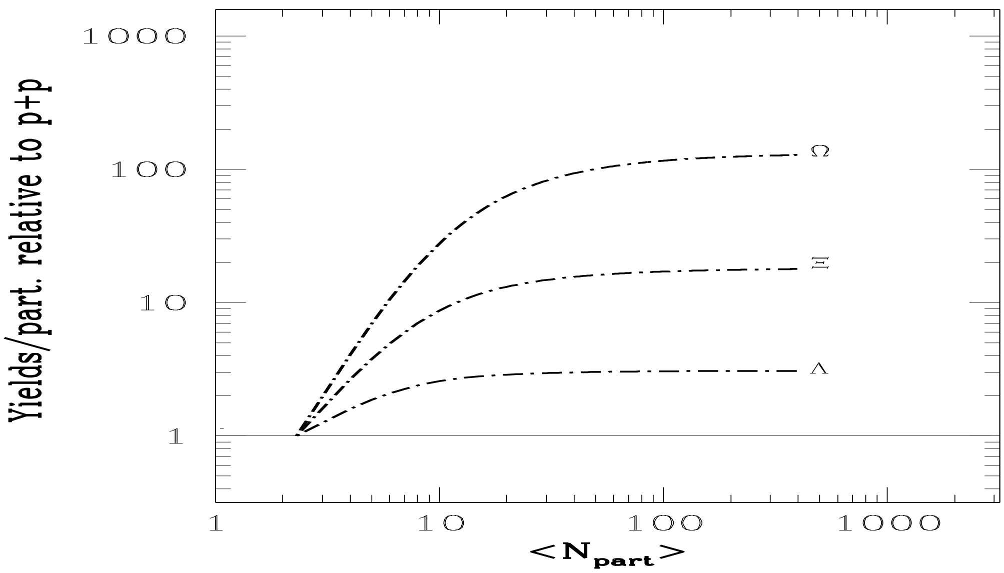


good quantitative description, nucl-th/0506044
But not the only one...

QGP enhancement or Canonical suppression

$$\lim_{V \to \infty} \frac{< N >_{CE}}{< N >_{GCE}} = 1$$

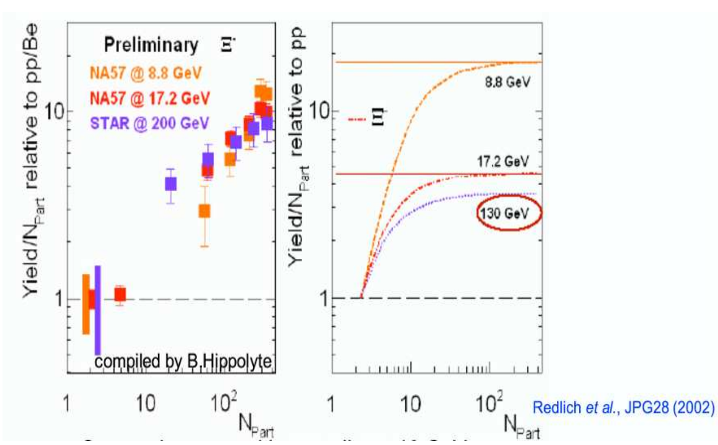
but away from thermodynamic limit$\longrightarrow$ additional suppression, nonlinear in volume (Hamieh,Tounsi,Becattini,Kera

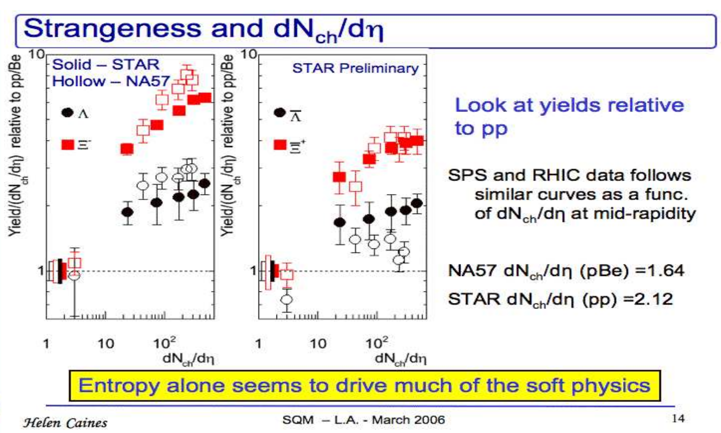


- Could strangeness enhancement be caused by the fact that p-p is far from the thermodynamic limit, while A-A is close to it? Is p-p particle production also governed by equilibrium statistics?

- Or could we be seeing 2 different production mechanisms, one (p-p) based on hadronic physics, the other one on QGP?
  (Hadronic transport models such as uRQMD can explain, without equilibrium p-p strangeness production but not A-A, e.g. NA57, Eur. Phys. J. C11 1999 79-88)

Energy and centrality dependance studies are increasingly challenging the canonical model

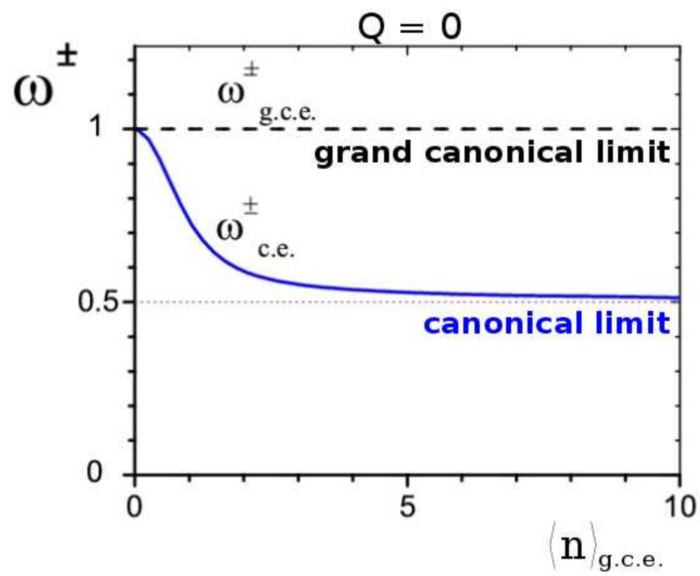




But could A definite falsification be carried out?

Second question: What ensemble most appropriate?
Fluctuations: The ensemble-O-meter

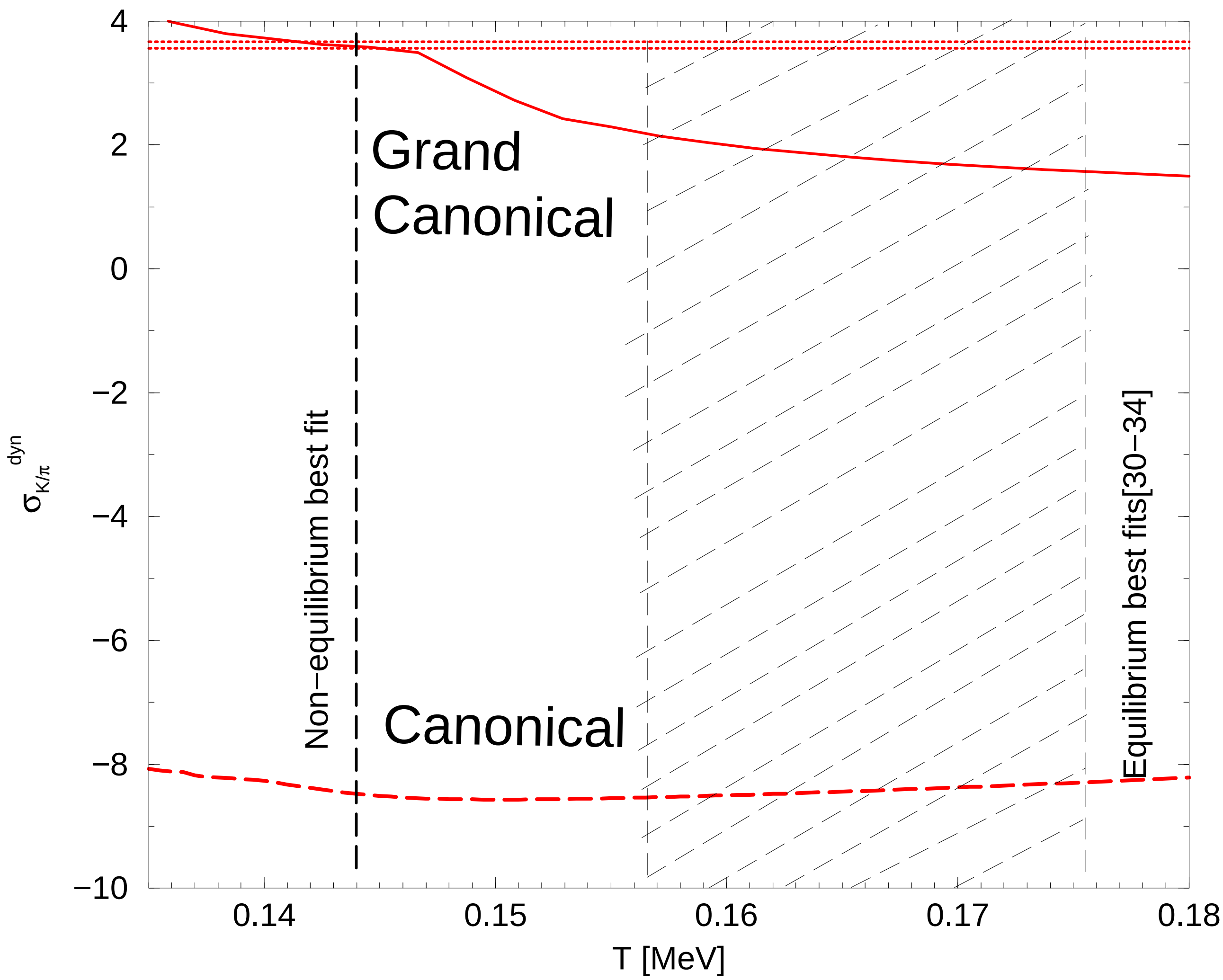
The dependance of fluctuations on yields is Ensemble-specific (Begun,Gorenstein,Gazdzicki,Zozulya)



It is very unlikely for the incorrect ensemble to describe both yields and fluctuations with the same parameters

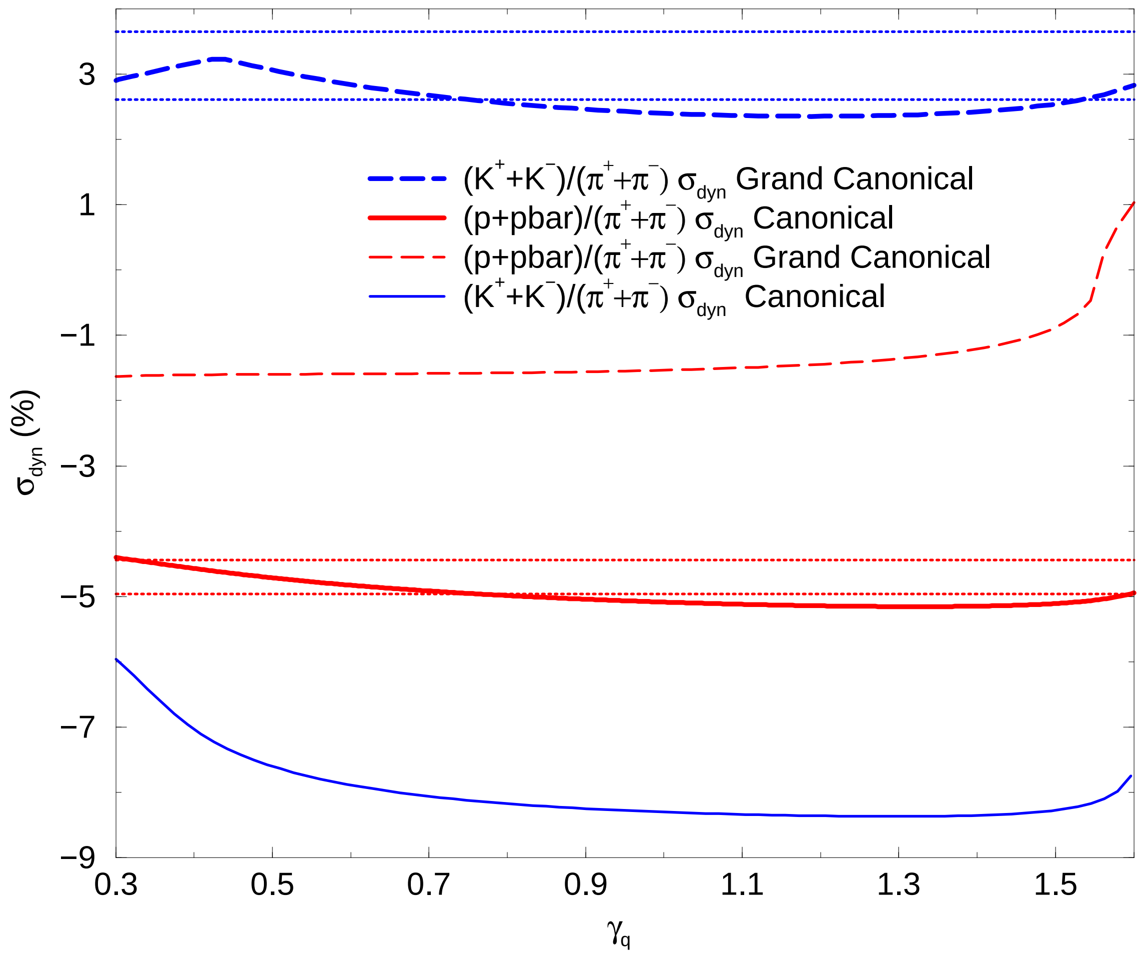
If canonical ensemble is a good description of strangeness in p-p collisions, than it has to describe strangeness fluctuations in p-p collisions with same T,V as yields

Let's try this: RHIC K/π fluctuations!
S. Das [STAR], hep-ex/0503023



Canonical ensemble has no hope of fitting preliminary K/π σ, Grand Canonical more or less OK (through need extra boost to fit well, more later).

But SPS $\sigma^{dyn}_{(p+\overline{p})/(\pi^{+}+\pi^{-})}$ is a different story!



Over-predicted by Grand canonical statistical model but works with Canonical ensemble for Baryon n.

I am still thinking about this...

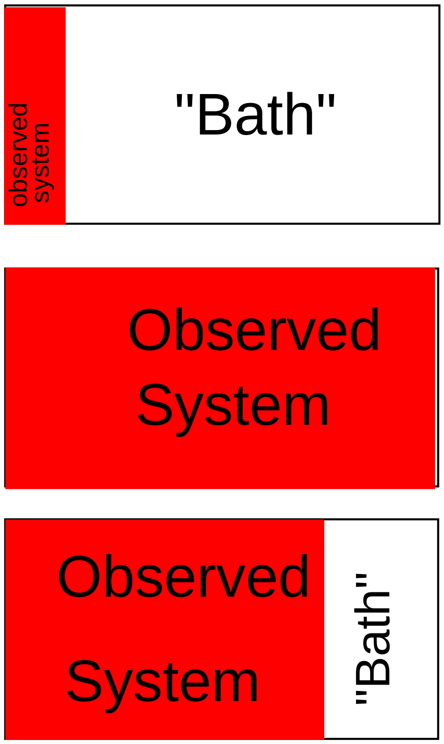
Why should baryon n. be Canonical and strangeness Grand Canonical?

Part II

Why quantitative studies of fluctuations can be dangerous

Fluctuations are a lot more prone to systematic distortions than yields. If we are going to use them to kill models based on experimental data, we have to be extra careful!

Global conservation laws



GC Ensemble $\omega_N \sim 1$

(+Resonances)

Local,not global Equilibrium

$\langle N \rangle = \langle N \rangle_{GC}$ $\quad \omega_N = 0$

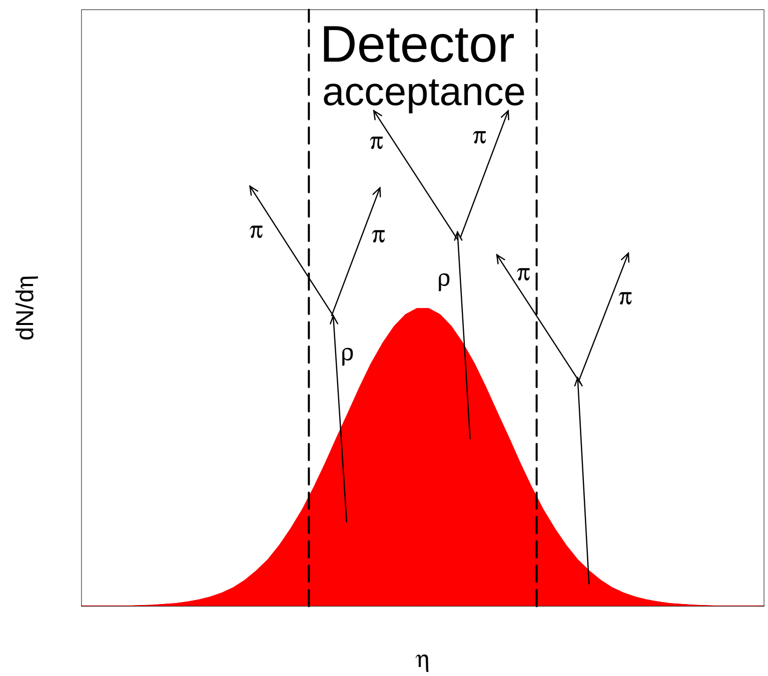
Conservation laws−> Long range correlations

$\omega_N = ??????$

Correction coefficient to Grand Canonical esemble (by expanding total enthropy around system number of particles)

$$\zeta_{GC} = \frac{\langle N \rangle}{2} \frac{(\partial^2 S/\partial N^2)_{N_{\rm tot}}}{(\partial S/\partial N)_{N_{\rm tot}}} \approx \frac{\eta_{exp}}{2\eta_{tot}} \left[\frac{\sum_{n=0}^{\infty} \lambda^n m^2 T K_2\left(\frac{nm}{T}\right)}{\ln\lambda \sum_{n=0}^{\infty} \lambda^n m^2 \frac{T}{n} K_2\left(\frac{nm}{T}\right)} \right]$$

GC description requires $\zeta_{GC} \ll 1$ ($\sim 13\%$ at STAR) subproblem III: Corrections to correlations due to limited acceptance

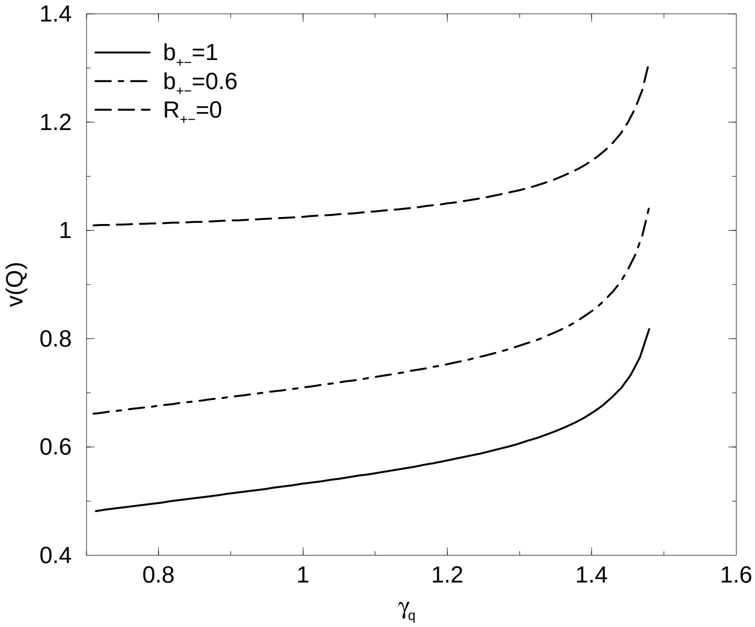


$\rho \longrightarrow N^{+}N^{-}$, but detector has limited acceptance. Need fraction of resonances whose decay products are still within acceptance region.

For 2-body decay $\rho \longrightarrow \pi^{+}\pi^{-}$ 3 fractions needed:

- b_{+} N. of positive decay products still in window
- b_{-} N. of negative decay products still in window
- b_{+-} N. of decay products both in window

Same type of arguments in direct reconstruction, except resonance need not be reconstructible



$$\left\langle (\Delta Q)^2 \right\rangle =$$

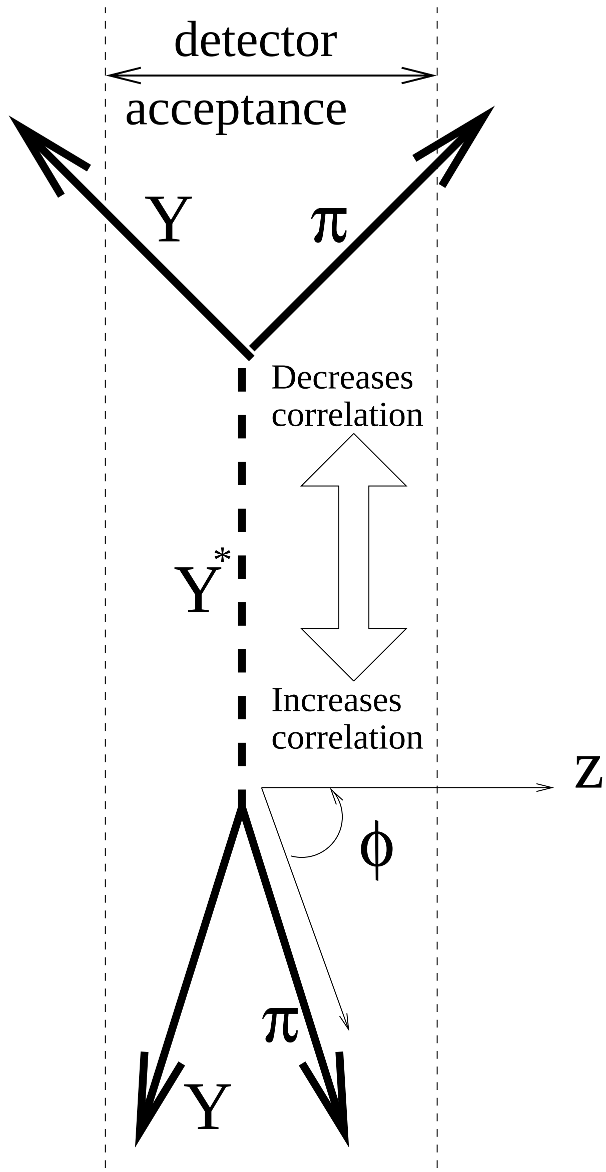
$$= \left\langle (\Delta N_+)^2 (b_+) \right\rangle + \left\langle (\Delta N_-)^2 (b_-) \right\rangle - 2 b_{+-} \left\langle \Delta N_+ \Delta N_- \right\rangle$$

Boost invariance: $b_+ = b_- = 1$ but $b_{+-} < 1$

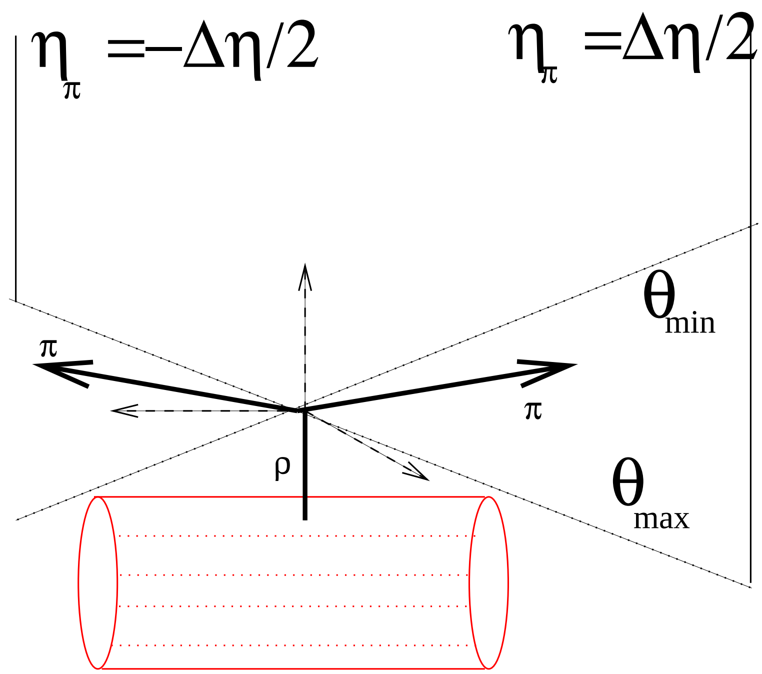
since p^* of $\rho \to N_+ N_-$ sets intrinsic rapidity scale!

To quantitatively extract T, γ_q,interaction time from fluctuations, b_{+-} has to be calculated for each resonance decay

Good news: Fluctuations still valid T_{chem} probe!



In local-thermal equilibrium Reactions destroying correlation and creating correlation balance out (again, up to $\sim \left\langle (\Delta C[f])^2 \right\rangle$). If physics local, even partial equilibrium should not destroy this balance.
But b_{+-} must still be calculated!



GT, S. Jeon, J. Rafelski, nucl-th/0503026

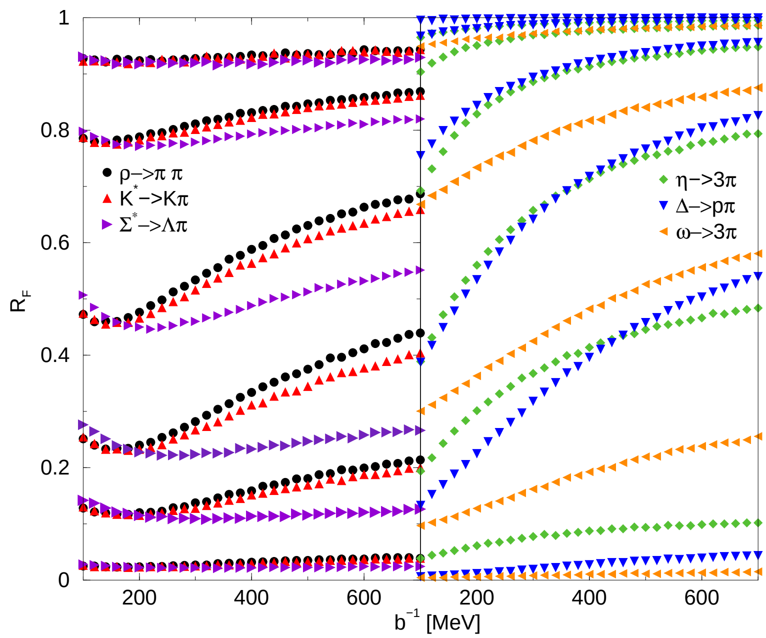
In a thermal-like source the fraction b_{+-} is given by a simple phase space integral

$$b_{+-} = \int_0^\infty dp_{TR} \int_{-\Delta\eta/2}^{\Delta\eta/2} d\eta_R P(\eta_R, p_{TR}) \Omega_{+-}(\eta_R, p_{TR})$$

$$\Omega_{+-}(\eta_R, p_{TR}) = \int \frac{d^3p_+^*}{E_+^*} \frac{d^3p_-^*}{E_-^*} \prod_i \frac{d^3p_i^*}{E_i^*} \Theta_{+-}$$

where:

$$\Theta_{+-} = \Theta_{\eta_+ - \frac{\Delta\eta}{2}} \Theta_{\eta_+ + \frac{\Delta\eta}{2}} \Theta_{\eta_- - \frac{\Delta\eta}{2}} \Theta_{\eta_- + \frac{\Delta\eta}{2}}$$



$$\frac{dN}{dy m_T dm_T} \propto e^{-b^{-1} m_T}$$

- Parameter b includes both temperature and flow

- It needs to be estimated at <u>chemical freeze-out</u>. It's possible since
  - Dependance on b small for most resonance decays
  - Re-interaction tends to increase flow and decrease T, so b not too affected

  Work in progress to put these on quantitative footing

Conclusions: Why fluctuations are good!

Fluctuations, taken together with yields, are a powerful tool of model differentiation. They are capable of:

- Falsifying all statistical models
- Determining experimentally the physically appropriate ensemble in the heavy ion regime
- Together with the direct detection of resonances, directly measure the effect of hadronic reinteractions between chemical and thermal freeze-out.
- Quantitatively determine
  - Freeze-out temperature
  - Non-equilibrium occupation parameters

  And experimentally distinguish between higher temperature equilibriu and super-cooled non-equilibrium freeze-out.

Conclusions: Issues to keep under control before comparing data to (statistical) models

- Experimental acceptance must be small for GC ensemble to be physically appropriate

- Correction coefficients for all leading resonance decays must be estimated

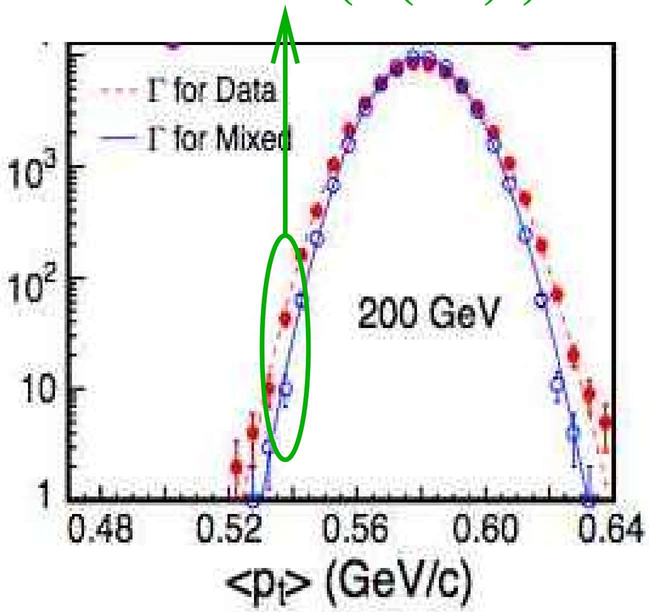
- Volume fluctuations must be kept under control (by choice of observables, fitting, or ansatz such as KNO).

Outlook:

SHAREv2.0

http://www.physics.arizona.edu/~torrieri/SHARE/share.html

Can this ($O(10^{-2})$)



Be as rich in insights as this (10^{-6})?