QCD Equation of State

in a Quasi-Particle Perspective

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- Comparison with Lattice QCD
- Features of CEP
- Family of Equations of State

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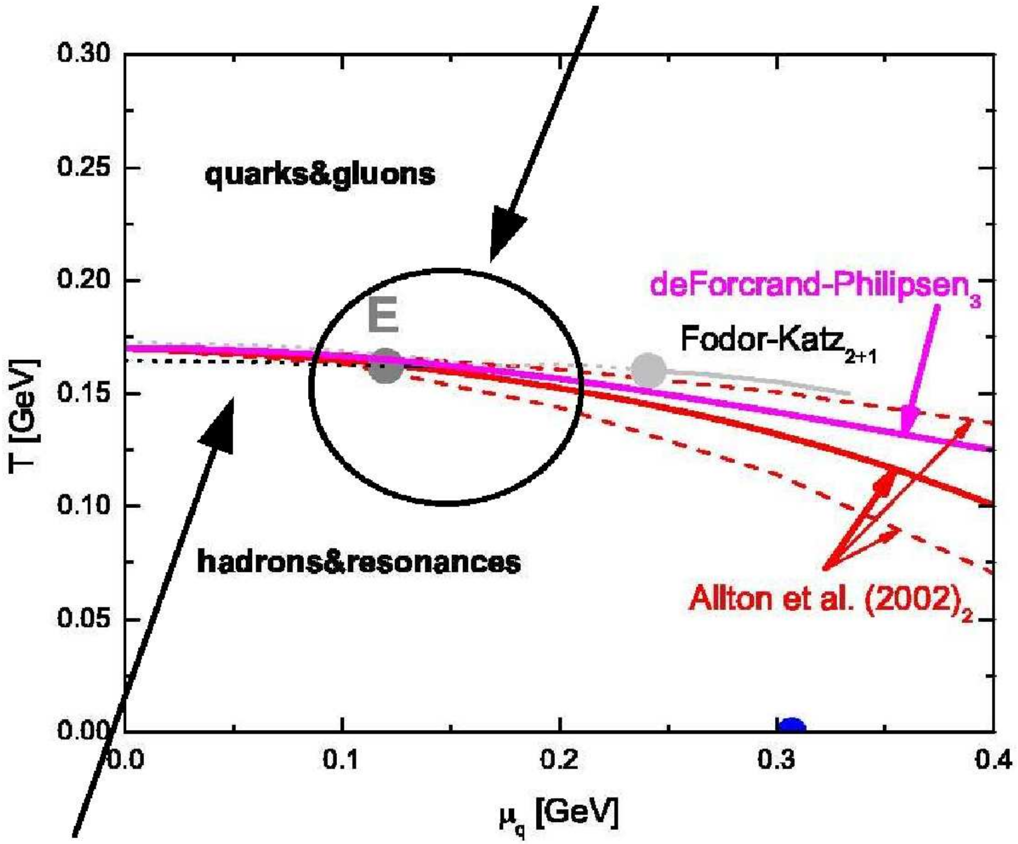
supported by BMBF, GSI

Motivation

CBM @ FAIR : $E_{beam} = 6 - 40$ AGeV



Universality hypothesis:

QCD Critical Point $\Rightarrow$
3D Ising Model

$(\mu_{B,E}, T_E) = (360, 165)$ MeV

no CEP - signatures in
current lattice QCD data
$\mu_B \leq 300$ MeV

- EoS: $p(T, \mu_q) = T^4 \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu_q}{T}\right)^{2n}$ $\Rightarrow$ hydrodynamics: RHIC

Allton et al. 2003, 2005: $N_f = 2$ – c_0, $c_{2,4,6}$

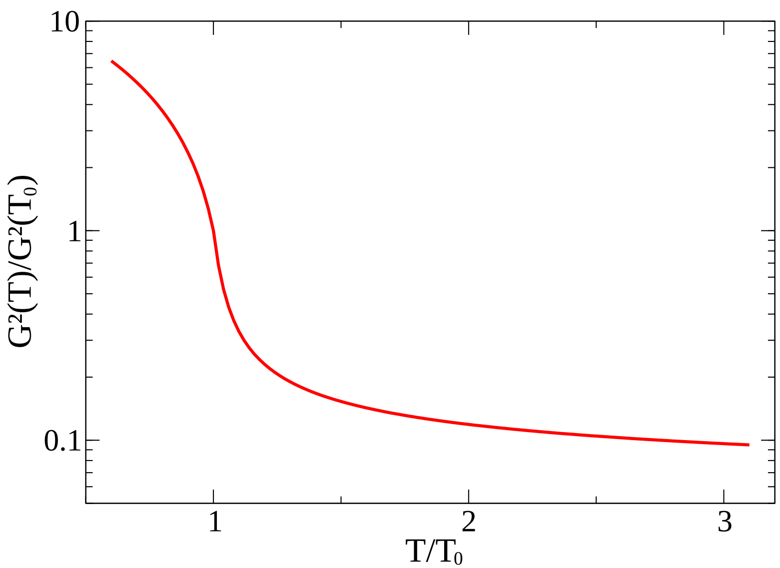
Ejiri et al. 2005: $N_f = 2$ – $\frac{dc_n}{dT}$ $\rightarrow$ e, s

Quasi-Particle Model

$$p(T,\mu_q) = \sum_{i=q,g} p_i(T,\mu_q;\Pi_i(T,\mu_q)) - B(\Pi_q,\Pi_g) \quad \rightarrow \quad c_n(T) = \frac{1}{n!}\frac{\partial^n p}{\partial \mu_q^n}\Big|_{\mu_q=0}$$

motivation: Luttinger - Ward approach to QCD $=$ propagator formalism
$\rightarrow$ chain of approximations: Φ - functional scheme

non-perturbative effects: replace $g^2 \rightarrow$ effective coupling at $\mu_q = 0$:



$$G^2(T) = \begin{cases} G^2_{2-\mathrm{loop}}(\zeta),\ \zeta = \lambda\frac{T-T_s}{T_c}, & T \geq T_c \\ G^2_{2-\mathrm{loop}}(T_c) + b(1-\frac{T}{T_c}), & T < T_c \end{cases}$$

lQCD data $\rightarrow$ change of G^2 at T_c

$\Longrightarrow$ "implemented phase transition"

$$\mu_q \neq 0: \quad a_T\frac{\partial G^2}{\partial T} + a_\mu\frac{\partial G^2}{\partial \mu} = b$$

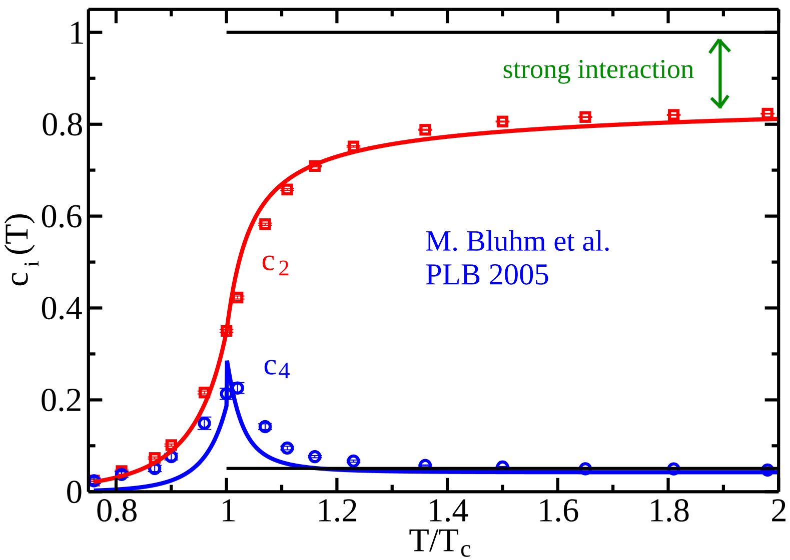
Peshier's flow equation

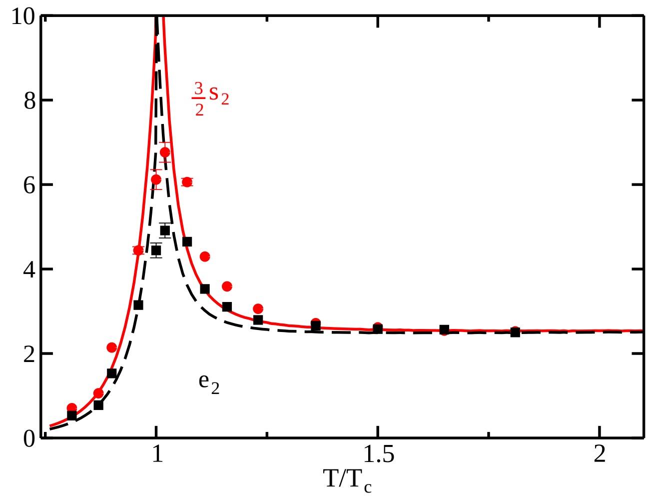
$G^2 \quad \rightarrow \quad$ pronounced structures in c_n

Energy and Entropy Density

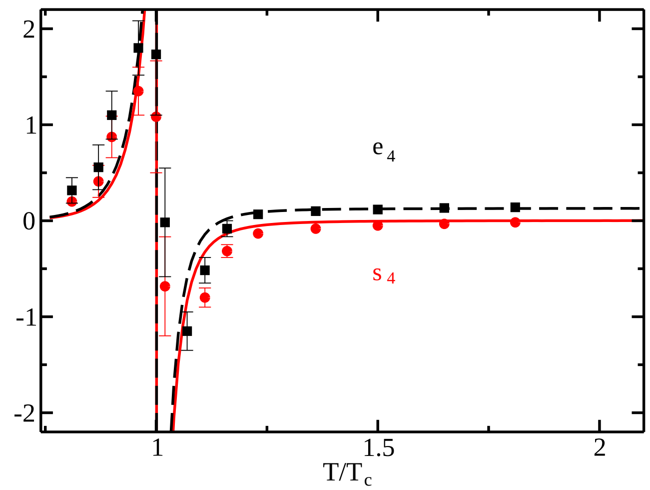
$$e(T,\mu_q) = T^4 \sum_{n=0}^{\infty} e_n(T) \left(\frac{\mu_q}{T}\right)^n \qquad s(T,\mu_q) = T^3 \sum_{n=0}^{\infty} s_n(T) \left(\frac{\mu_q}{T}\right)^n$$

$$\begin{aligned} e_n &= 3c_n + Tc_n' \\ s_n &= (4-n)c_n + Tc_n' \end{aligned}$$





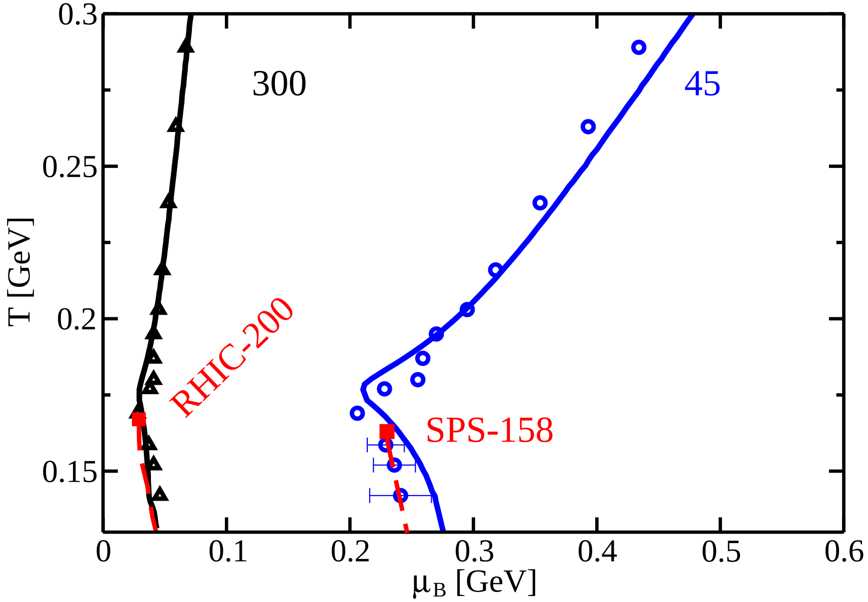
more sensitive test ⟵



Isentropic EoS - $N_f = 2$

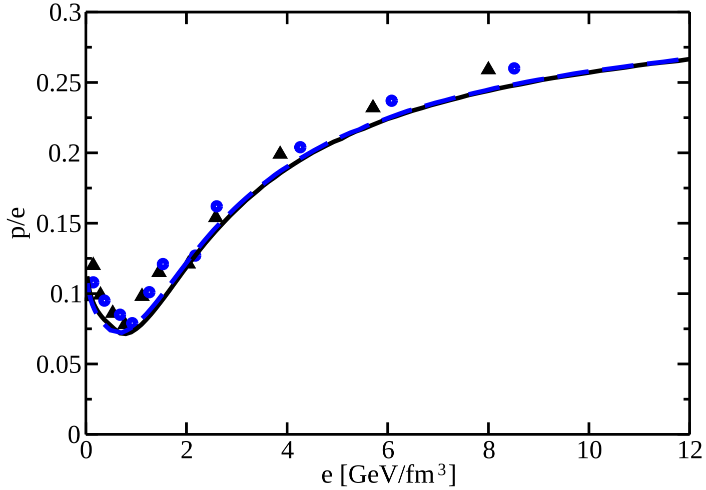
$$n_B(T,\mu_B) = T^3 \sum_{n=0}^{\infty} 2n c_{2n}(T) \left(\frac{\mu_B}{3T}\right)^{2n-1}$$

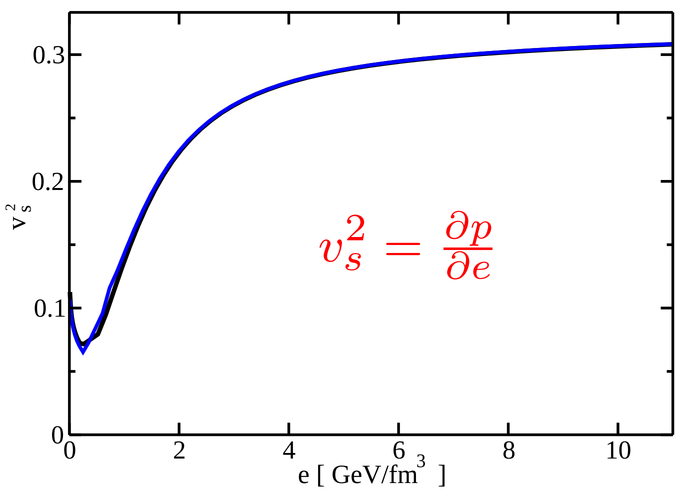
$$\rightarrow s/n_B = \text{const.}$$



Ejiri et al. 2005

$\Rightarrow$ hints for consistency with chemical freeze-out points





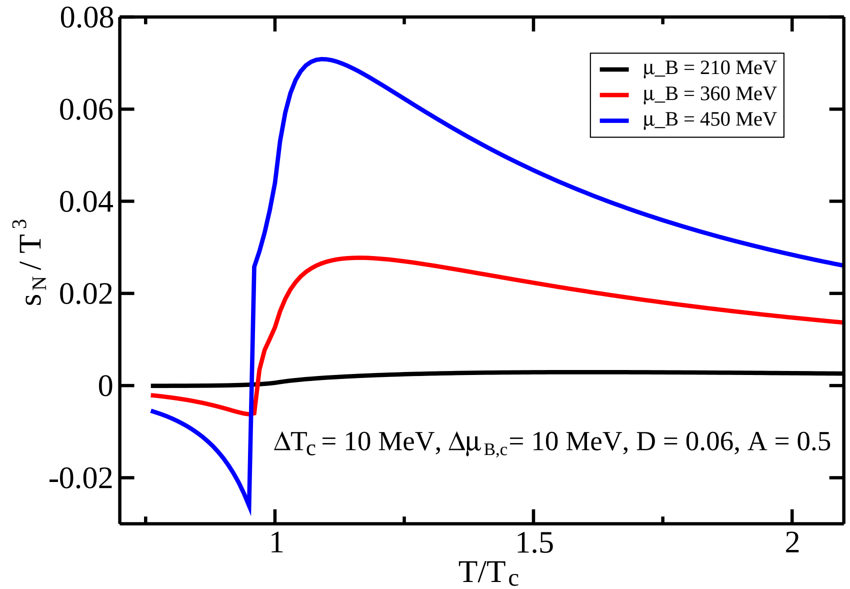
$\Rightarrow$ almost independent of s/n_B

CEP - Features

estimated phase boundary:

$$T_c(\mu_B) = T_c\left(1 + \frac{c}{18}\left(\frac{\mu_B}{T_c}\right)^2\right)$$

$c = -0.14(6)$ Allton et al. 2002
$c = -0.122$ QPM



entropy density: $s = s_{QPM} + s_N$

$$s_N(T, \mu_B) = \frac{2}{9} c_2(T) \mu_B^2 T A \tanh S_c(T, \mu_B)$$

$\longrightarrow$ non-analytic, small for small μ_B

$$S_c(T, \mu_B) = -D\left(\Delta T_c^2 + \Delta\mu_{B,c}^2\right)^{1/2}\left(\frac{\partial G}{\partial T}\right)_{\mu_B}$$

$\longleftarrow$ Gibbs' free enthalpy G
(Guida, Zinn-Justin)

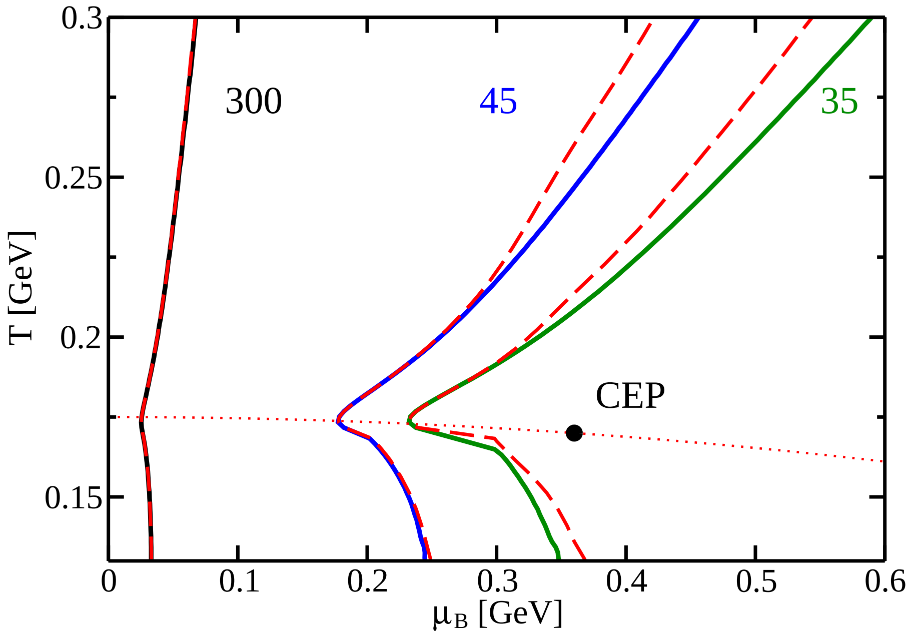
(Nonaka, Asakawa)

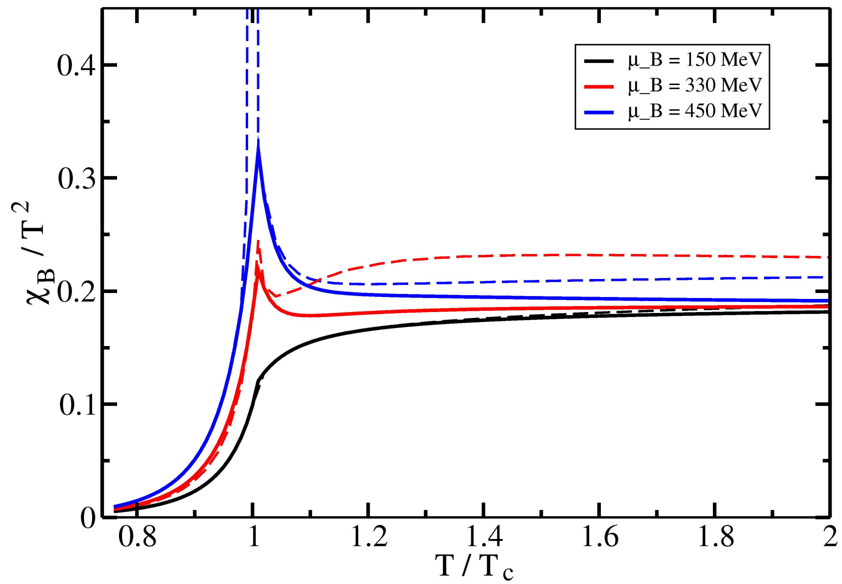
$\longrightarrow$ critical region: $\Delta\mu_{B,c}$, ΔT_c small

CEP Influence on Isentropic EoS - $N_f = 2$

baryon number susceptibility:

$$\chi_B = \frac{\partial n_B}{\partial \mu_B}$$





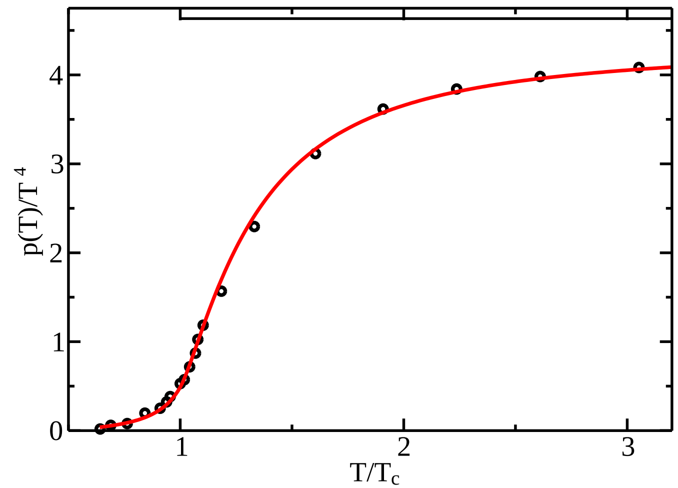
→ generic pattern

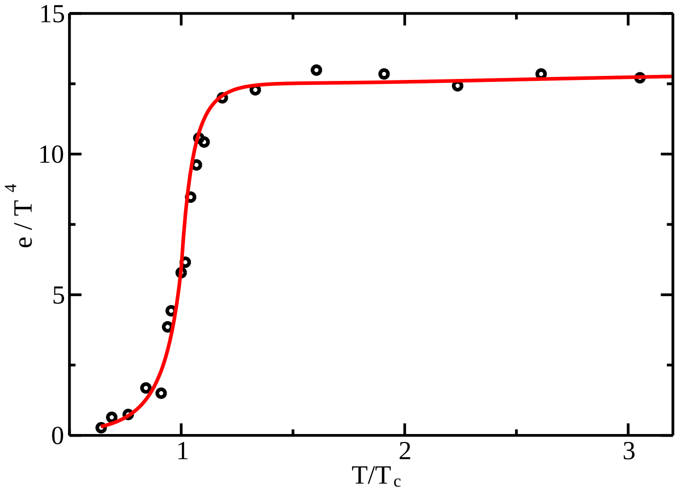
measure for fluctuations

→ singular for $\mu_B \geq \mu_{B,E}$

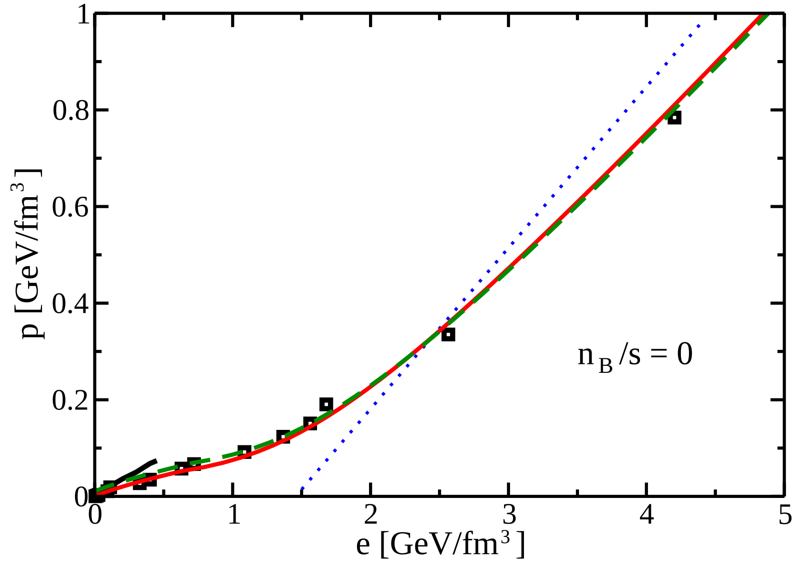
EoS - $N_f = 2 + 1$

$p(e, n_B) \quad \rightarrow$ hydrodynamics





RHIC: hydro-evolution independent of CEP

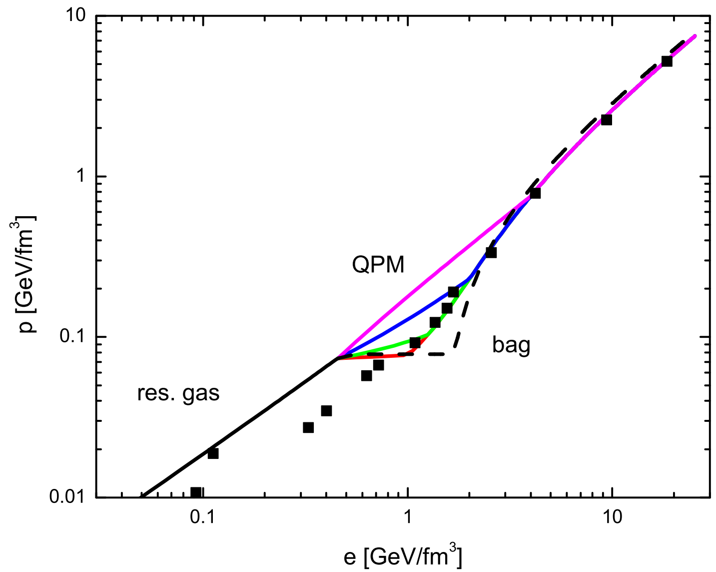


chiral extrapolation: naive $m_{0,q} = 0$ and $m_{0,s} = 150$ MeV

n_B - dependence from flow equation tiny for RHIC: $n_B \leq 0.4$ fm^{-3}

Karsch et al. 2003

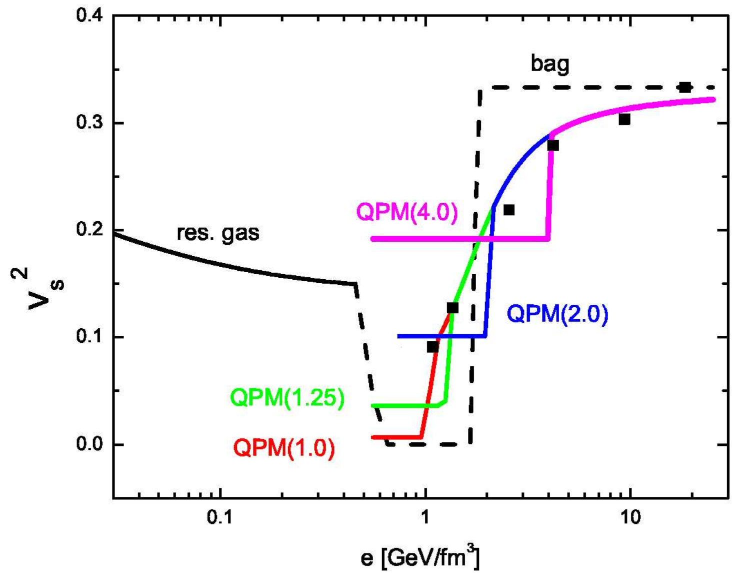
Family of EoS



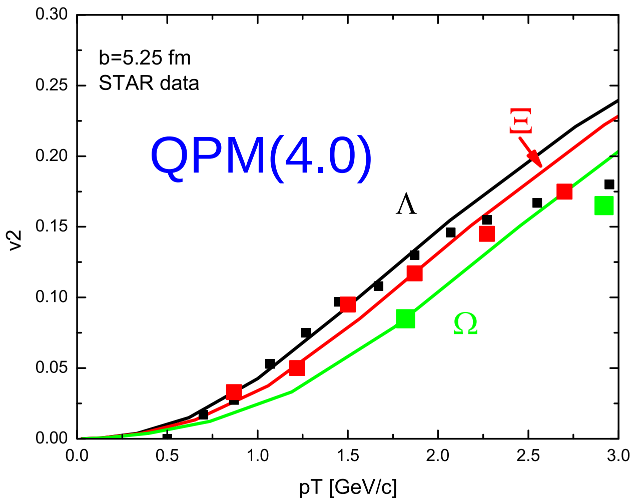
←high density part fixed: lattice QCD

linear interpolation better than extrapolation

←low density part fixed: resonance gas



Hydrocode - P. Kolb → Hyperons: $v_2(p_\perp)$



Conclusions

- QPM describes lattice QCD data: $N_f = 2,\ 2+1$
- $N_f = 2$: Taylor expansion coefficients $c_n(T)$

  more sensitive $\rightarrow\ e_n(T)$, $s_n(T)$

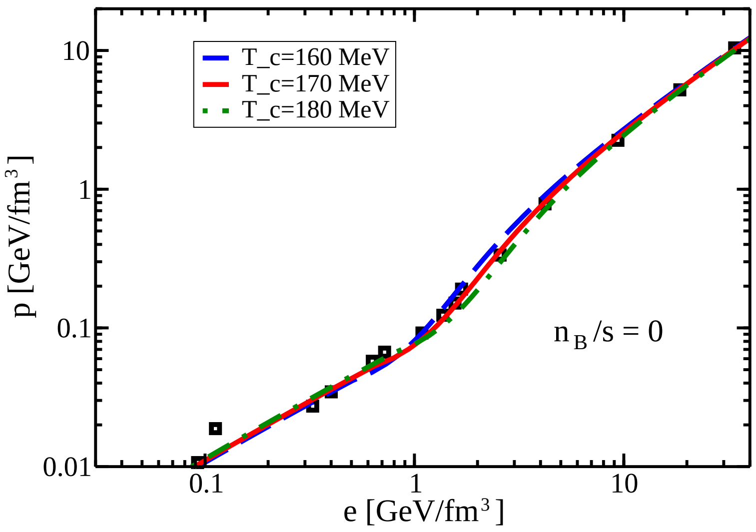
  $G^2 \rightarrow$ pronounced structures $\equiv$ test of Peshier's flow equation
- Isentropic Equation of State: $N_f = 2,\ 2+1$

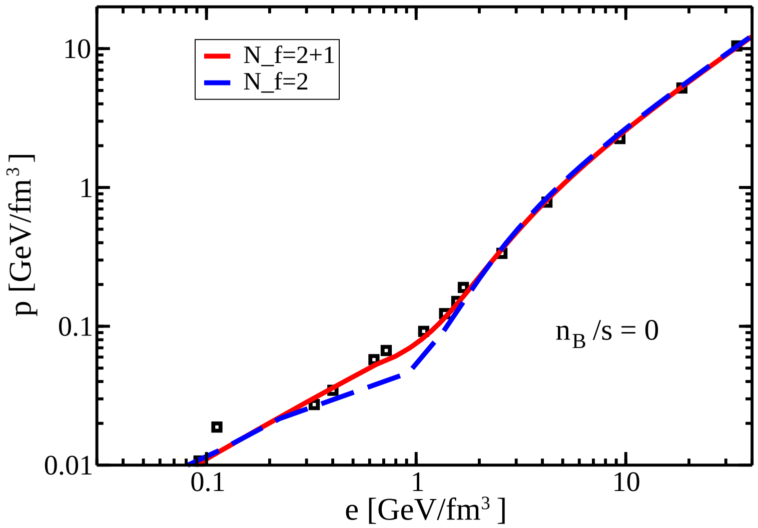
  pattern of trajectories, consistency with chemical freeze-out points
- CEP-Inclusion: many new parameters

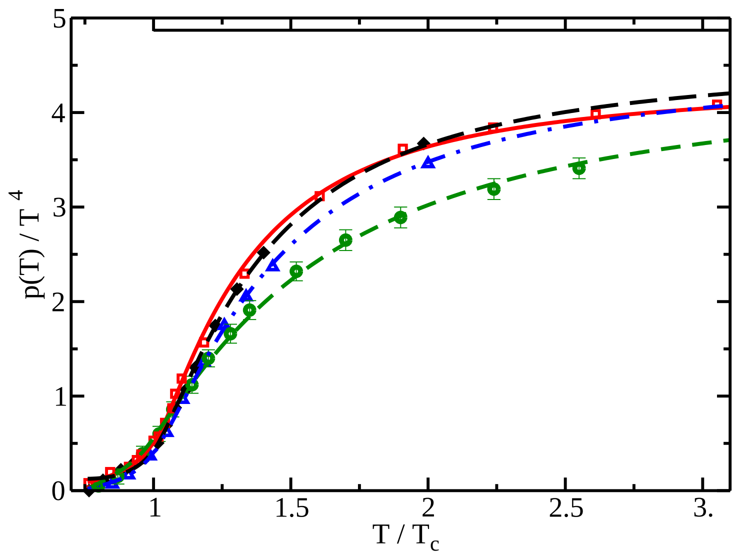
  generic pattern of isentropic trajectories unchanged

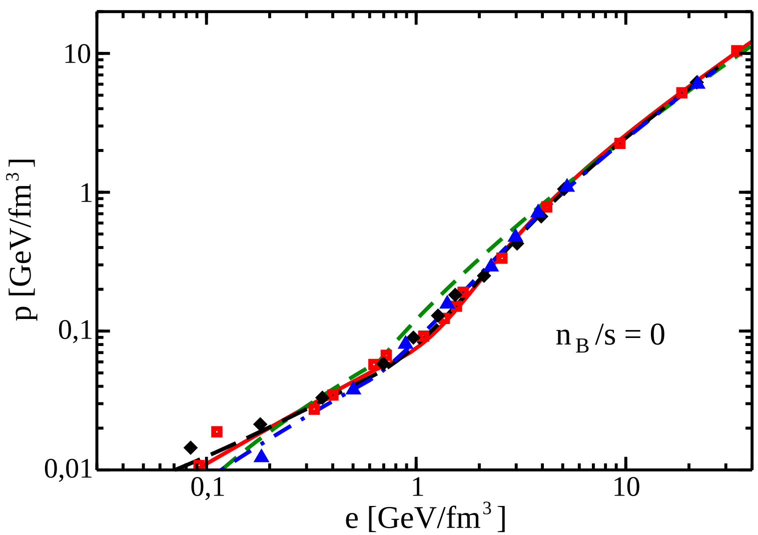
  singular behavior in fluctuations: χ_B
- provide Family of EoS $\rightarrow$ hydrodynamics: $v_2(p_\perp) \leftarrow$ RHIC
- CERN SPS, CBM-FAIR: CEP and baryon number effects on EoS

Stability of EoS



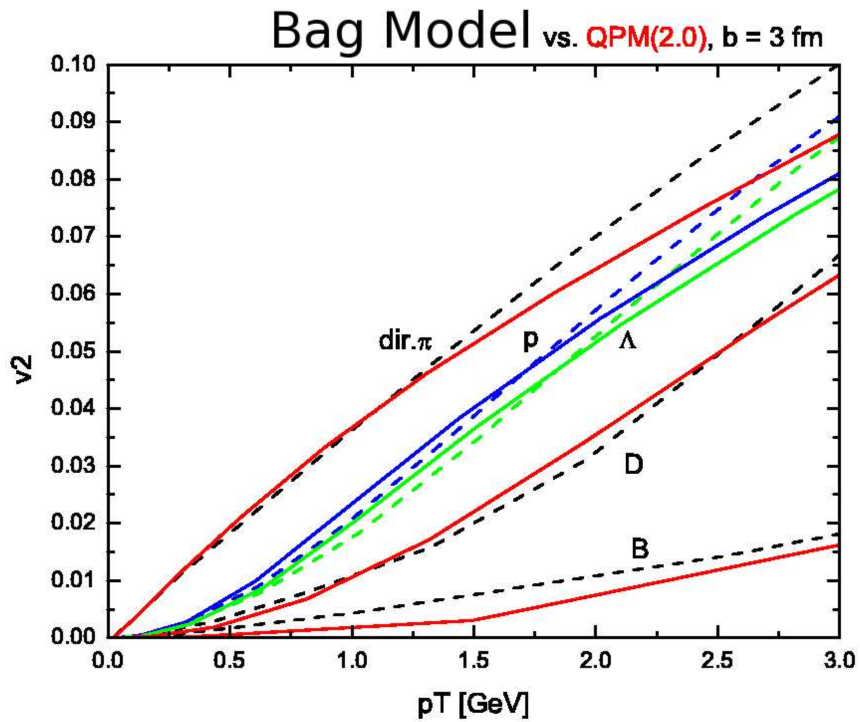


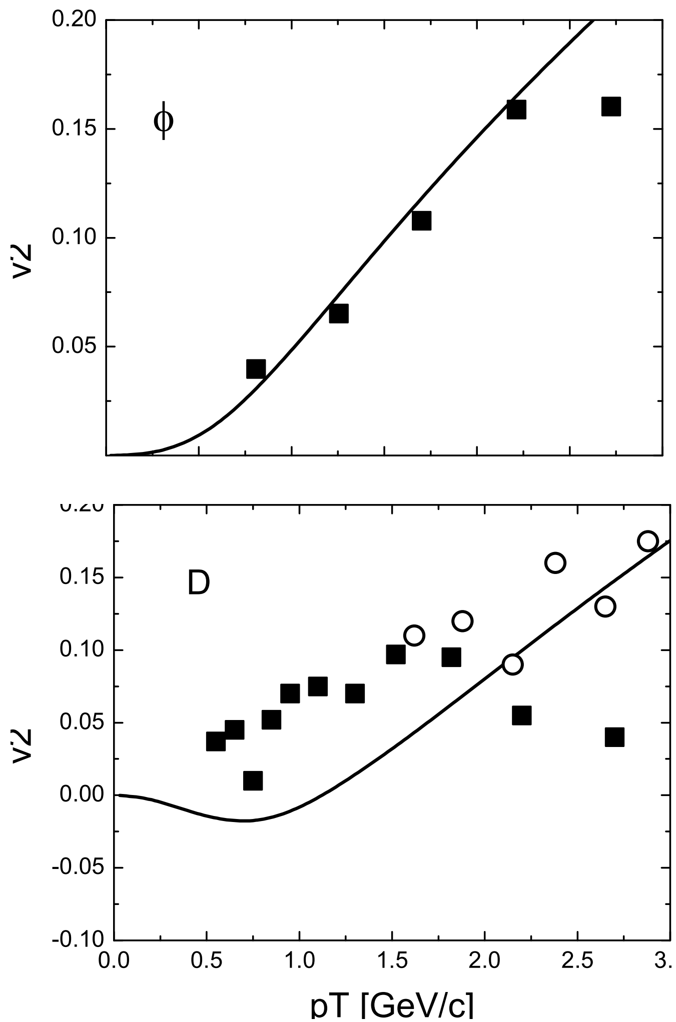




More on elliptic flow

EoS must be sufficiently soft in transition region





Comparison of different EoS