

Phases of QCD

Claudia Ratti

ECT, Trento, ITALY and Technical University, Munich, GERMANY*

In collaboration with Simon Rößner, Michael A. Thaler and Wolfram Weise

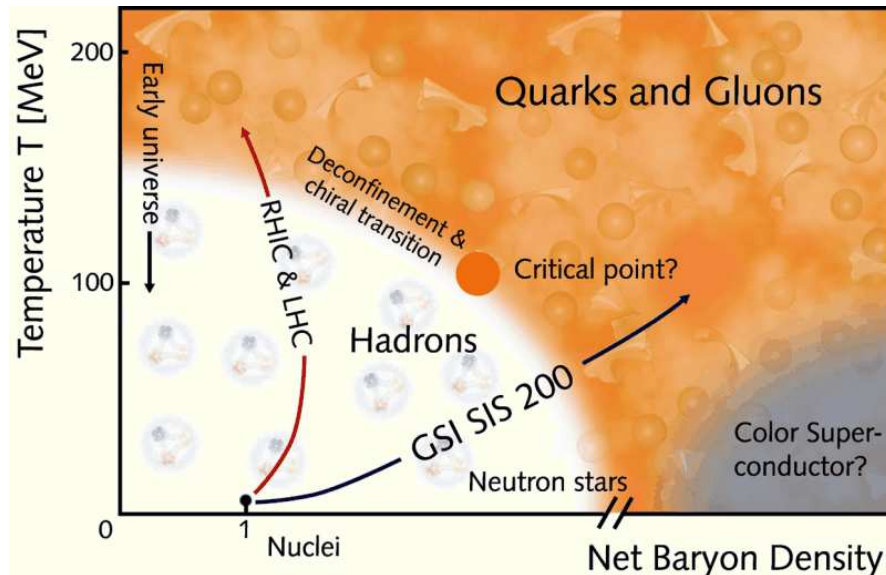
Can results of
Lattice QCD Thermodynamics
be understood in terms of
QUASIPARTICLE
degrees of freedom?

$$N_f = 2$$

$$N_f = 2 + 1$$

$$N_f = 3$$

Introduction



- ❖ QCD has a rich phase structure
- ❖ Many challenging items:
 - ➡ order of the phase transition
 - ➡ critical point
 - ➡ deconfinement and chiral symmetry
 - ➡ colour superconductivity at high μ

❖ Status of **lattice QCD thermodynamics**:

- ➡ precise data available in the pure gauge sector
- ➡ quarks easily introduced at $\mu = 0$
- ➡ first lattice data at finite (small) μ (F. Karsch, Z. Fodor, S. Katz, P. de Forcrand, O. Philipsen, M. D'Elia, M. P. Lombardo).

PNJL (Polyakov loop extended NJL) model

Starting point: three-flavour NJL model in temporal background gauge field

NJL four-fermion interaction

$$\mathcal{L}_{PNJL} = \bar{\psi} (i\gamma_\mu D^\mu - \hat{m}_0) \psi + \frac{G}{2} \sum_{f=u,d,s} \left[(\bar{\psi}_f \psi_f)^2 + (\bar{\psi}_f i\gamma_5 \vec{\lambda} \psi_f)^2 \right] \\ - \frac{K}{2} \left[\det_{i,j} (\bar{\psi}_i (1 + \gamma_5) \psi_j) + \det_{i,j} (\bar{\psi}_i (1 - \gamma_5) \psi_j) \right] - V(\Phi, T),$$

NJL six-fermion interaction

PNJL (Polyakov loop extended NJL) model

Starting point: three-flavour NJL model in temporal background gauge field

$$\hat{m}_0 = \text{diag} \{m_u, m_u, m_s\}$$

NJL four-fermion interaction

$$\mathcal{L}_{PNJL} = \bar{\psi} (i\gamma_\mu D^\mu - \hat{m}_0) \psi + \frac{G}{2} \sum_{f=u,d,s} \left[(\bar{\psi}_f \psi_f)^2 + (\bar{\psi}_f i\gamma_5 \vec{\lambda} \psi_f)^2 \right] \\ - \frac{K}{2} \left[\det_{i,j} (\bar{\psi}_i (1 + \gamma_5) \psi_j) + \det_{i,j} (\bar{\psi}_i (1 - \gamma_5) \psi_j) \right] - V(\Phi, T),$$

NJL six-fermion interaction

PNJL (Polyakov loop extended NJL) model

Starting point: three-flavour NJL model in temporal background gauge field

$$D_\mu = \partial_\mu - iA_\mu ; A_\mu = \delta_{\mu 0} A_0$$

$$\hat{m}_0 = \text{diag} \{ m_u, m_u, m_s \}$$

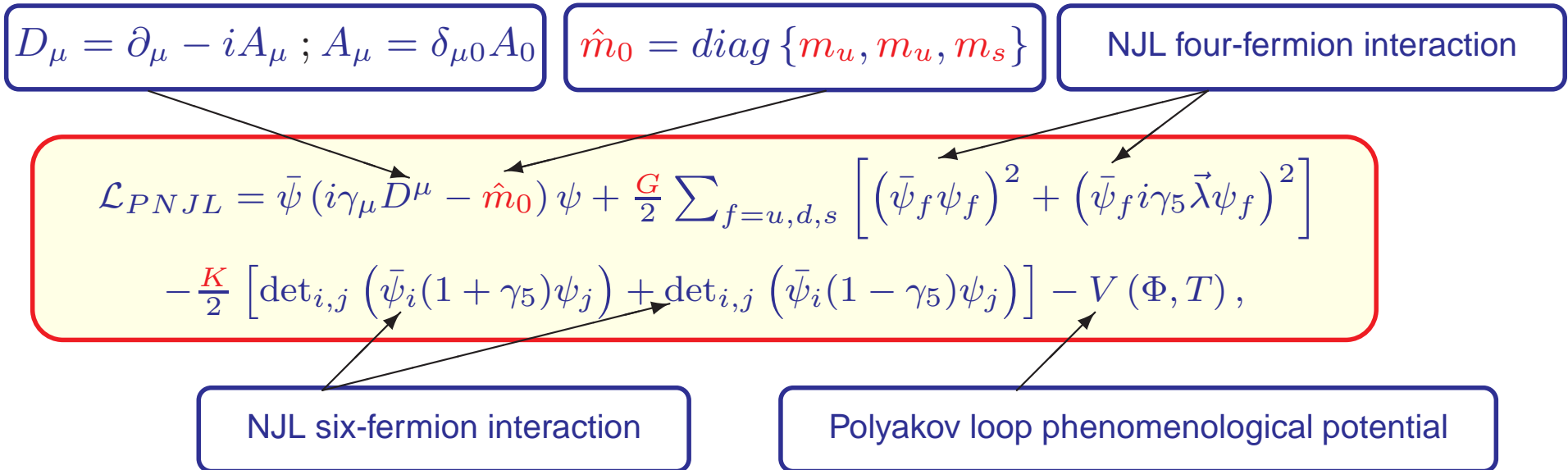
NJL four-fermion interaction

$$\mathcal{L}_{PNJL} = \bar{\psi} (i\gamma_\mu \bar{D}^\mu - \hat{m}_0) \psi + \frac{G}{2} \sum_{f=u,d,s} \left[(\bar{\psi}_f \psi_f)^2 + (\bar{\psi}_f i\gamma_5 \vec{\lambda} \psi_f)^2 \right] \\ - \frac{K}{2} \left[\det_{i,j} (\bar{\psi}_i (1 + \gamma_5) \psi_j) + \det_{i,j} (\bar{\psi}_i (1 - \gamma_5) \psi_j) \right] - V(\Phi, T),$$

NJL six-fermion interaction

PNJL (Polyakov loop extended NJL) model

Starting point: three-flavour NJL model in temporal background gauge field



Coupling between Polyakov loop and quarks **uniquely determined** by covariant derivative D_μ . We recall that:

$$\Phi = \frac{1}{N_c} \text{Tr} \left[\mathcal{P} \exp \left(i \int_0^\beta A_4 d\tau \right) \right], \quad A^0 = -iA_4.$$

Parameters: m_u, m_s, G, K, Λ fixed in the hadronic sector.

Polyakov loop potential

- ❖ The Polyakov loop is the **order parameter** related to the $Z(N_c)$ symmetry

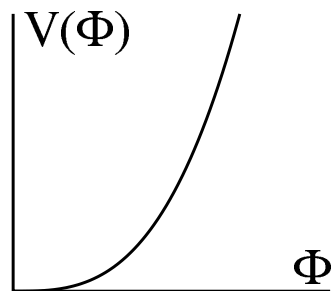
$$\frac{V(\Phi, T)}{T^4} = -\frac{b_2(T)}{2} \Phi^* \Phi - \frac{b_3}{6} (\Phi^3 + (\Phi^*)^3) + \frac{b_4}{4} (\Phi^* \Phi)^2$$

with

$$b_2(T) = a_0 + a_1 \left(\frac{T_0}{T}\right) + a_2 \left(\frac{T_0}{T}\right)^2 + a_3 \left(\frac{T_0}{T}\right)^3$$

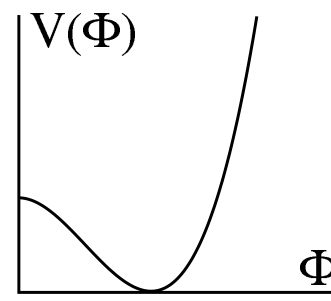
$$T < T_c$$

- color confinement
- $\langle \Phi \rangle = 0 \rightarrow Z(3)$ unbroken



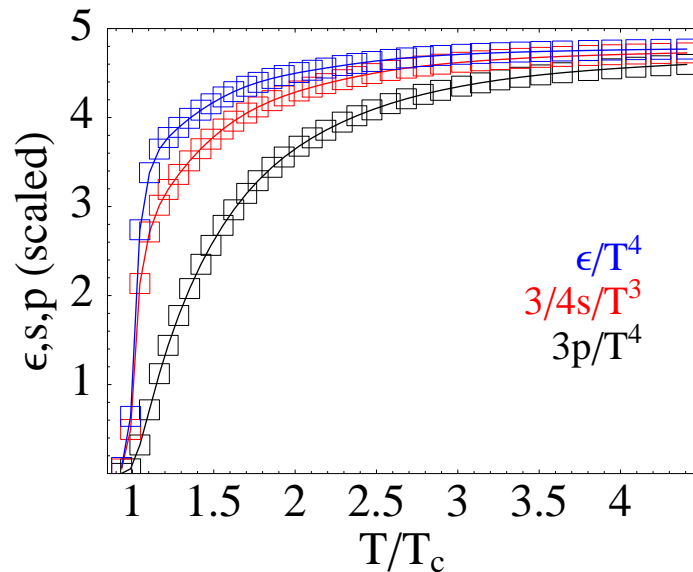
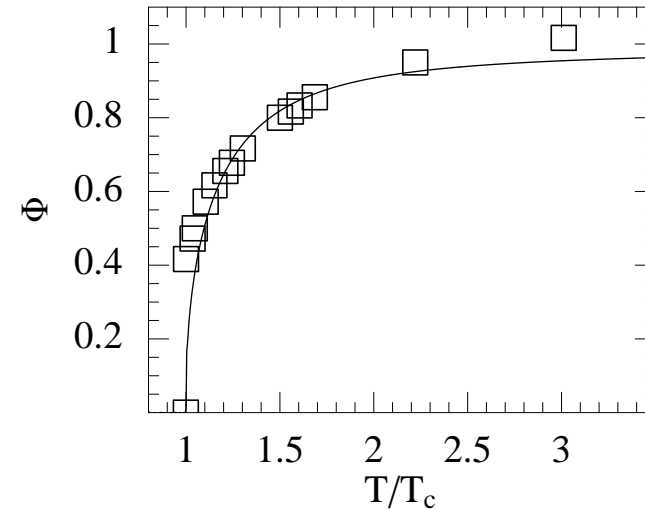
$$T > T_c$$

- color deconfinement
- $\langle \Phi \rangle \neq 0 \rightarrow Z(3)$ broken



Fit of Pure Gauge QCD lattice data

- ◆ Minimization of $V(\Phi, T)$: Polyakov loop behaviour as a function of T
- ◆ Comparison with lattice data from [Kaczmarek *et al.* PLB 543 \(2002\)](#)



- ◆ $p(T) = -V(\Phi(T), T)$
- ◆ $s(T) = \frac{dp}{dT} = -\frac{dV(\Phi(T), T)}{dT}$
- ◆ $\epsilon(T) = T \frac{dp}{dT} - p = Ts(T) - p(T)$
- ◆ Comparison with lattice data from [Boyd *et al.* NPB 469 \(1996\)](#)

PNJL model at finite temperature and chemical potential

The thermodynamic potential of the system is:

$$\Omega(T, \mu) = V(\Phi, T) + \frac{\sigma_{u,d}^2}{2G} + \frac{\sigma_s^2}{4G} - \frac{K}{4G^3} \sigma_{u,d}^2 \sigma_s - 2 \sum_f T \int \frac{d^3 p}{(2\pi)^3} \left\{ \text{Tr}_c \ln \left[1 + L e^{-(E_{p,f} - \mu_f)/T} \right] \right. \\ \left. + \text{Tr}_c \ln \left[1 + L^\dagger e^{-(E_{p,f} + \mu_f)/T} \right] + 3 \frac{E_{p,f}}{T} \theta(\Lambda^2 - \vec{p}^2) \right\}.$$

with $E_{p,f} = \sqrt{p^2 + M_f^2}$ and $\text{Tr}_c L = \Phi$, $\text{Tr}_c L^\dagger = \Phi^*$.

Interaction with chiral condensate: quarks develop a **constituent** mass:

$$M_i = m_i - \langle \sigma_i \rangle - \frac{K}{4G^2} \langle \sigma_j \rangle \langle \sigma_k \rangle = m_i - 2G \langle \bar{\psi}_i \psi_i \rangle + K \langle \bar{\psi}_j \psi_j \rangle \langle \bar{\psi}_k \psi_k \rangle \quad i \neq j \neq k$$

Minimization of Ω : behaviour of

❖ Quark masses (chiral condensates)

❖ Polyakov loop

as functions of

❖ temperature

❖ quark chemical potential.

Final form for Ω 1-quark (antiquark) states, suppressed below T_c 2-quark (antiquark) states, suppressed below T_c

$$\Omega(T, \mu, \sigma, \Phi) = V(\Phi, T) + \frac{\sigma_{u,d}^2}{2G} + \frac{\sigma_s^2}{4G} - \frac{K}{4G^3} \sigma_{u,d}^2 \sigma_s^2$$

$$- 2 \sum_f \int \frac{d^3 p}{(2\pi)^3} \left\{ 3E_p + T \left[\ln \left[1 + 3\Phi e^{-(E_p - \mu)/T} + 3\Phi^* e^{-2(E_p - \mu)/T} + e^{-3(E_p - \mu)/T} \right] \right. \right.$$

$$\left. \left. + \ln \left[1 + 3\Phi^* e^{-(E_p + \mu)/T} + 3\Phi e^{-2(E_p + \mu)/T} + e^{-3(E_p + \mu)/T} \right] \right] \right\}$$

3-quark (antiquark) states, not suppressed even below T_c High temperature limit: $\Phi \rightarrow 1, \Phi^* \rightarrow 1$

We re-obtain the standard NJL formula:

$$\ln \left[1 + 3\Phi e^{-(E_p - \mu)/T} + 3\Phi^* e^{-2(E_p - \mu)/T} + e^{-3(E_p - \mu)/T} \right]$$

$$\downarrow T \rightarrow \infty$$

$$\ln \left[1 + e^{-(E_p - \mu)/T} \right]^3 = 3 \ln \left[1 + e^{-(E_p - \mu)/T} \right]$$

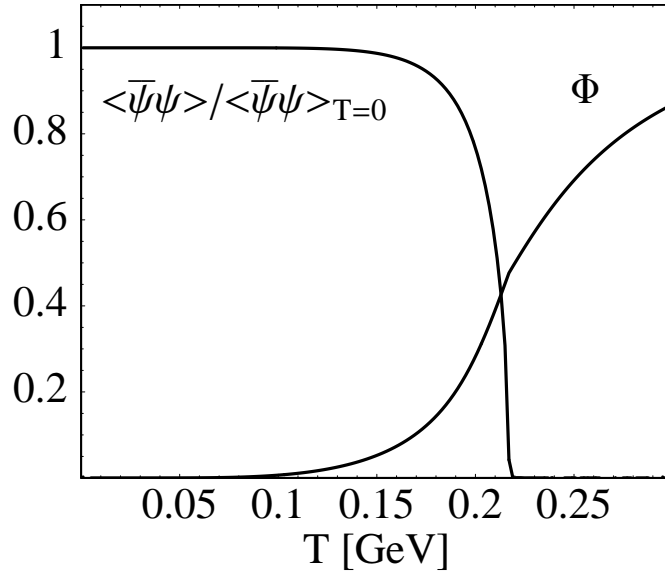
$$N_f = 2 \Leftrightarrow m_s = \infty$$

Lattice vs. PNJL model

Chiral and Deconfinement transitions

Zero and finite chemical potential

Confinement and chiral symmetry breaking: chiral limit ($m_u = 0$)



$T_c \simeq 270$ MeV in pure gauge

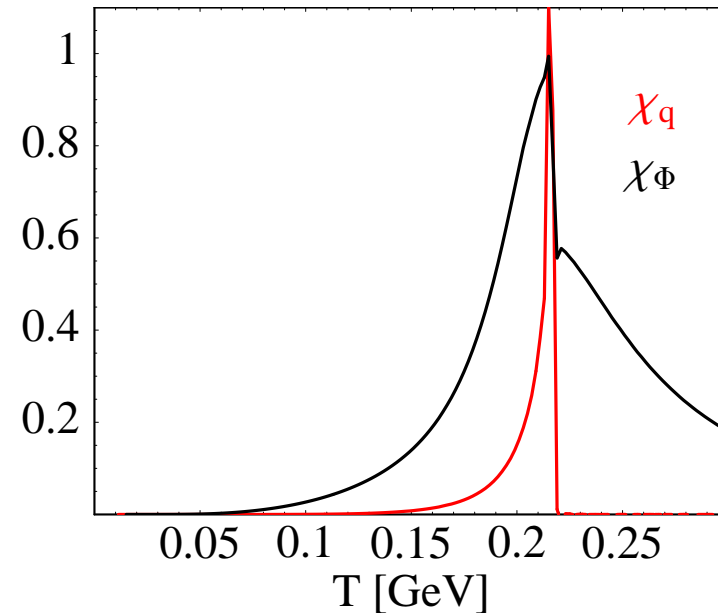


$T_c \simeq 210$ MeV with quarks

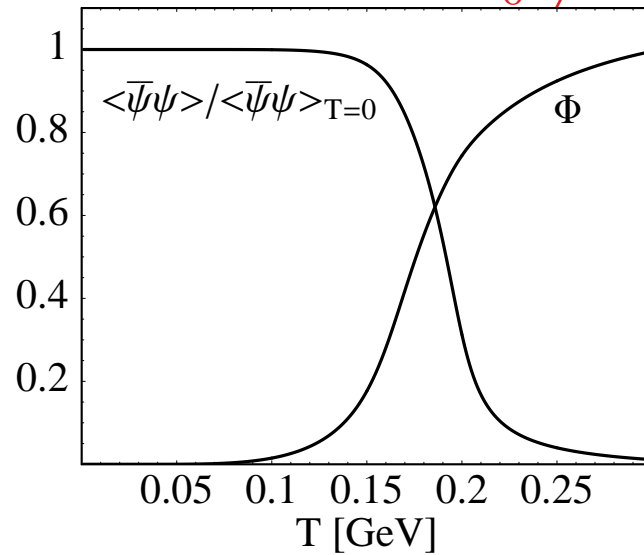
CHIRAL and DECONFINEMENT

transitions coincide in the

CHIRAL LIMIT!



$m_0 \neq 0, \mu = 0$ predictions



$T_c \simeq 210$ MeV

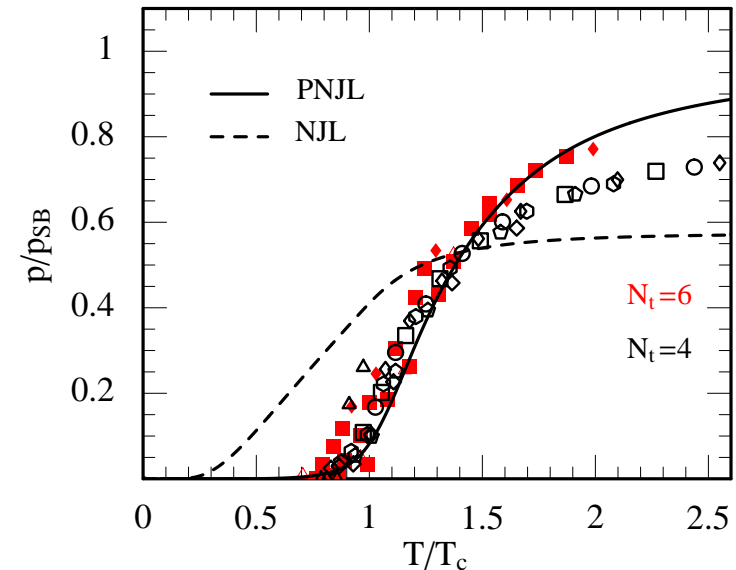


$T_c \simeq 170$ MeV

- ◆ Scaled pressure as a function of T/T_c

$$\frac{p(T, \mu = 0)}{T^4} = - \frac{\Omega(T, \mu = 0)}{T^4}$$

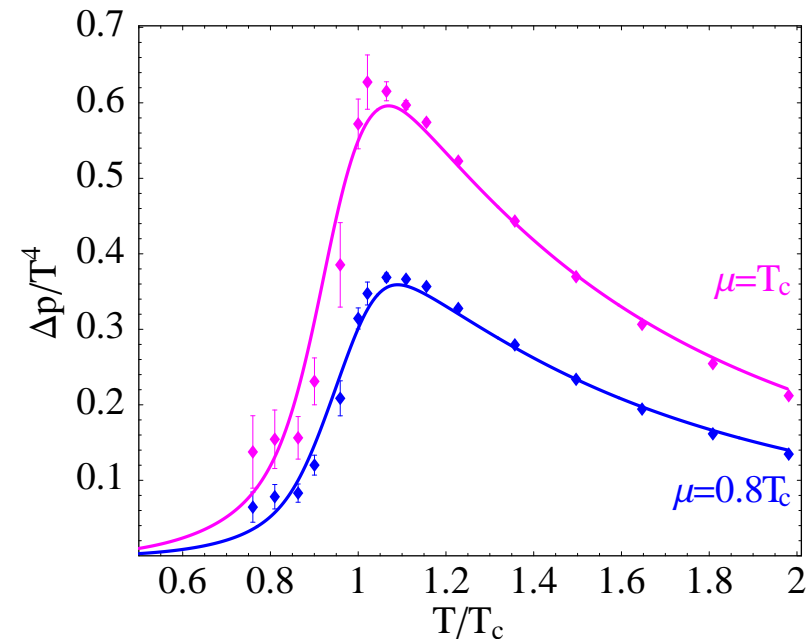
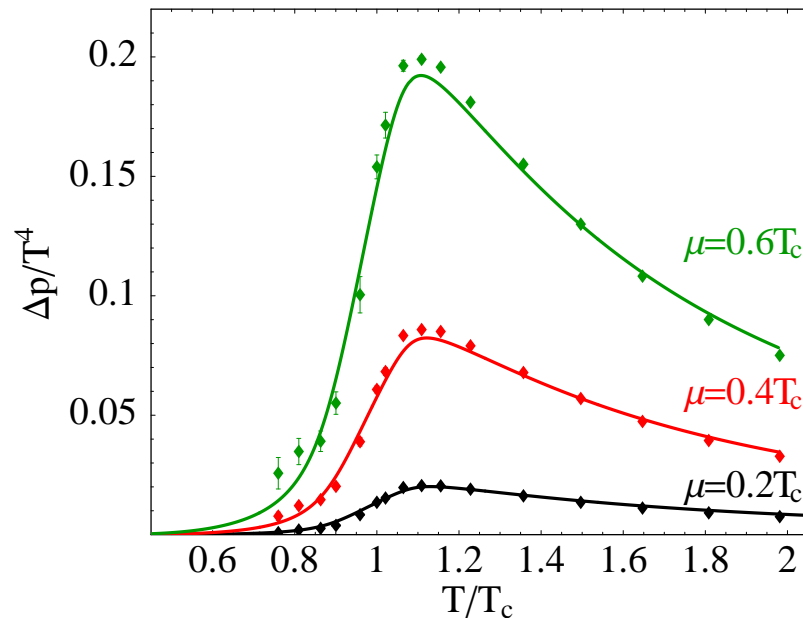
- ◆ Comparison with lattice data from CP-PACS collaboration (2001)



Finite μ predictions: pressure difference

- ◆ Scaled pressure difference as a function of T/T_c

$$\frac{\Delta p(T, \mu)}{T^4} = \frac{p(T, \mu) - p(T, \mu = 0)}{T^4}$$

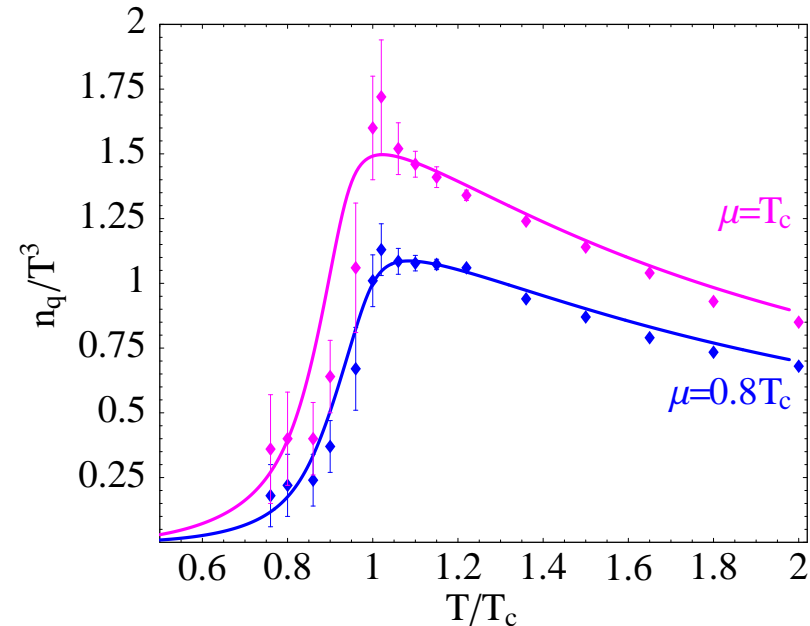
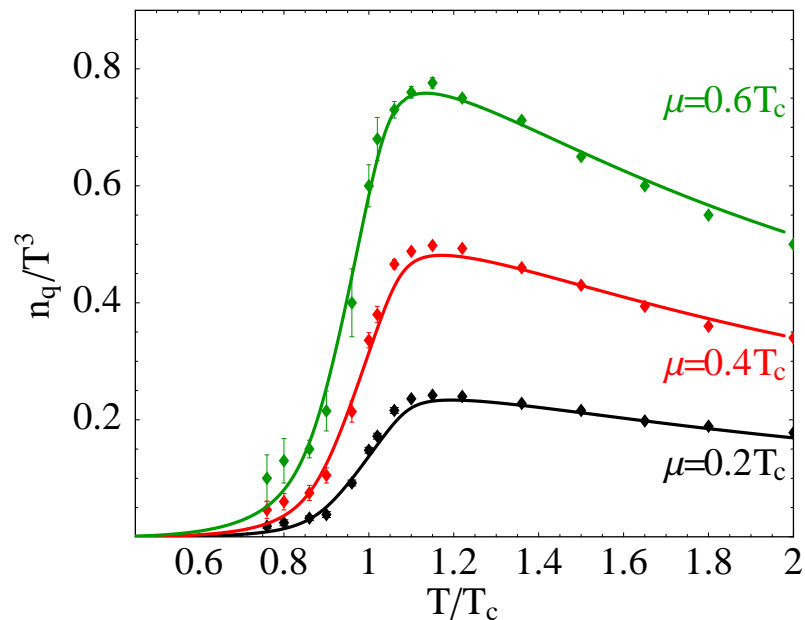


Scaled pressure difference as a function of the temperature at different values of the chemical potential, compared to the corresponding lattice data taken from [Allton *et al.*, PRD 68 \(2003\)](#). PNJL model results from: [C.R., M. A. Thaler and W. Weise, PRD 73 \(2006\)](#).

Finite μ predictions: quark number density

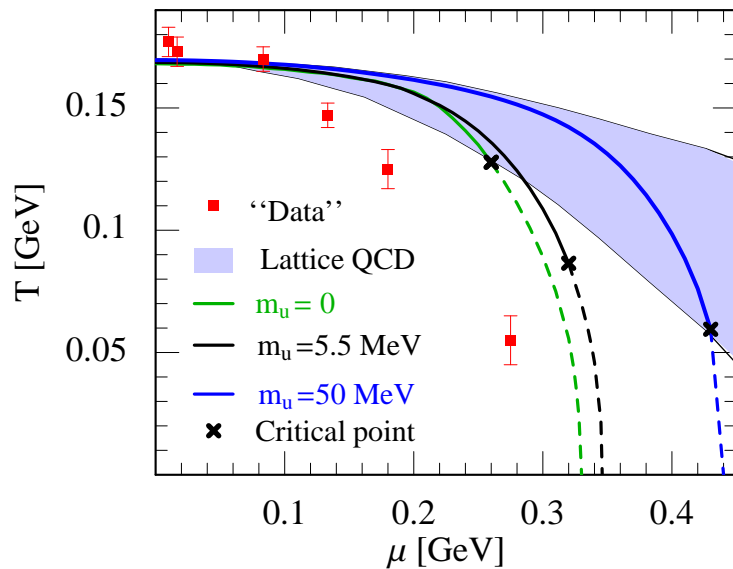
- ◆ Scaled quark number density as a function of T/T_c

$$\frac{n_q(T, \mu)}{T^3} = -\frac{1}{T^3} \frac{\partial \Omega(T, \mu)}{\partial \mu}$$

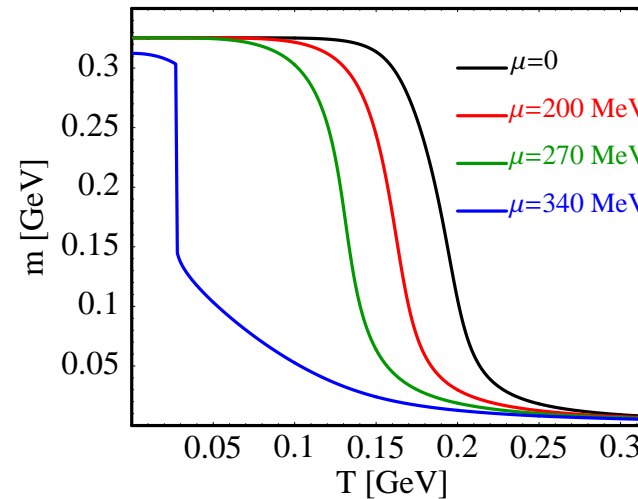


Scaled quark number density as a function of the temperature at different values of the chemical potential, compared to the corresponding lattice data taken from [Allton *et al.*, PRD 68 \(2003\)](#). PNJL model results from: [C.R., M. A. Thaler and W. Weise, PRD 73 \(2006\)](#).

Towards the PHASE DIAGRAM ($N_f = 2$)



Constituent quark mass

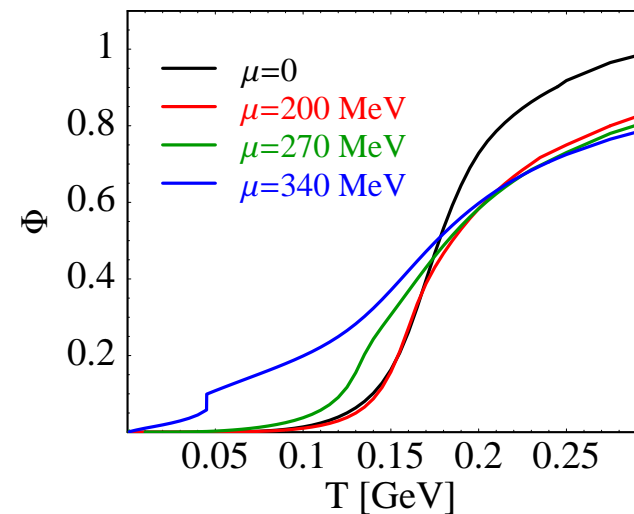


Therm. fit data from Andronic and Braun-Munzinger (2004)
 Lattice data from Allton *et al.* (2002)

**First order transition
 at large chemical potential!**

C. R., S. Rößner, W. Weise, in preparation

Polyakov loop



$$N_f = 2 + 1 \Leftrightarrow m_s \neq m_u$$

Lattice vs. PNJL model

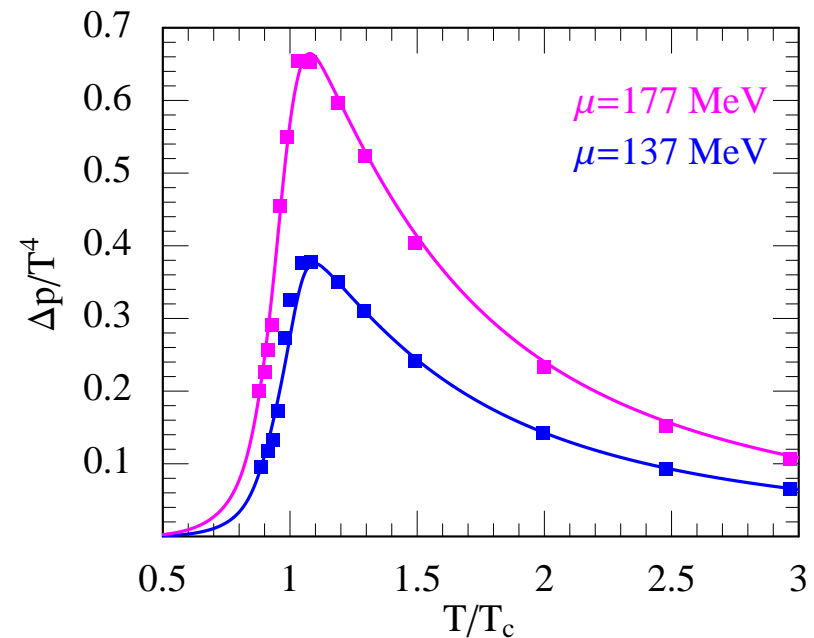
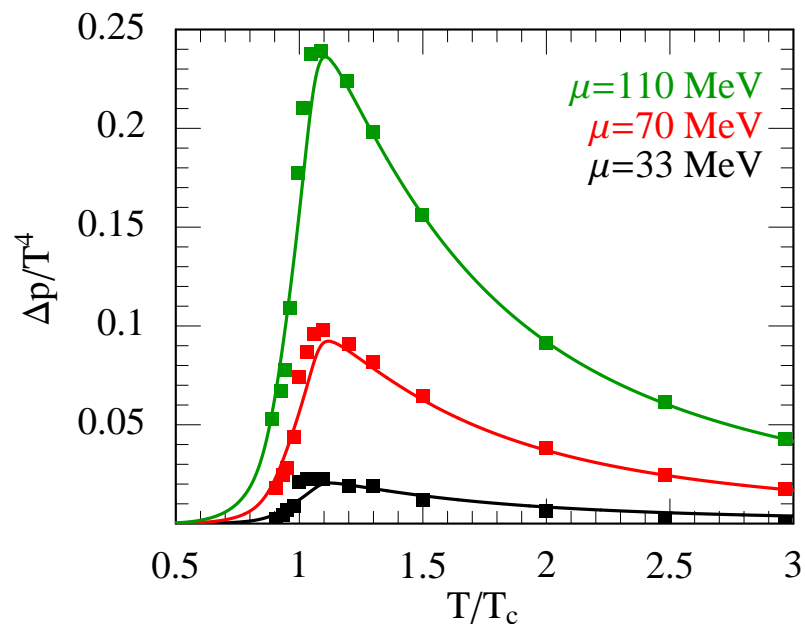
Finite chemical potential

C. R., M. A. Thaler and W. Weise, in preparation

Finite μ predictions: pressure difference ($\mu_s = 0$)

- ◆ Scaled pressure difference as a function of T/T_c

$$\frac{\Delta p(T, \mu)}{T^4} = \frac{p(T, \mu) - p(T, \mu = 0)}{T^4}$$

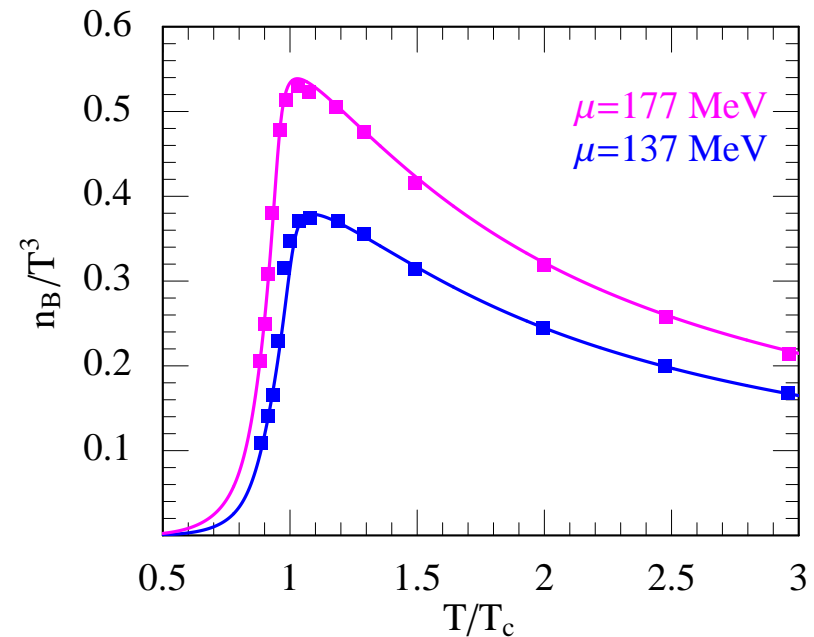
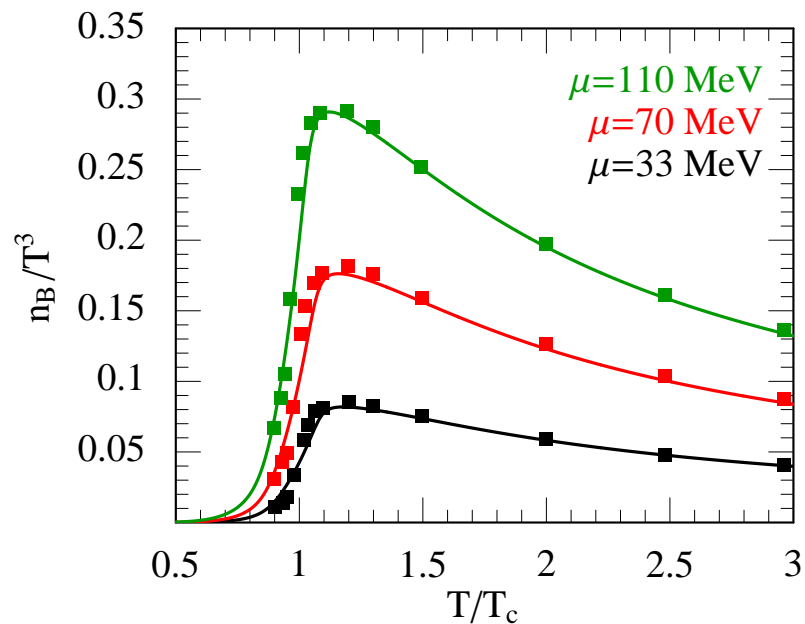


Scaled pressure difference as a function of the temperature at different values of the chemical potential, compared to the corresponding lattice data taken from [Z. Fodor *et al.*, PLB 568 \(2003\)](#).

Finite μ predictions: quark number density ($\mu_s = 0$)

- ◆ Scaled quark number density as a function of T/T_c

$$\frac{n_B(T, \mu)}{T^3} = -\frac{1}{3} \frac{1}{T^3} \sum_f \frac{\partial \Omega(T, \mu_f)}{\partial \mu_f}$$



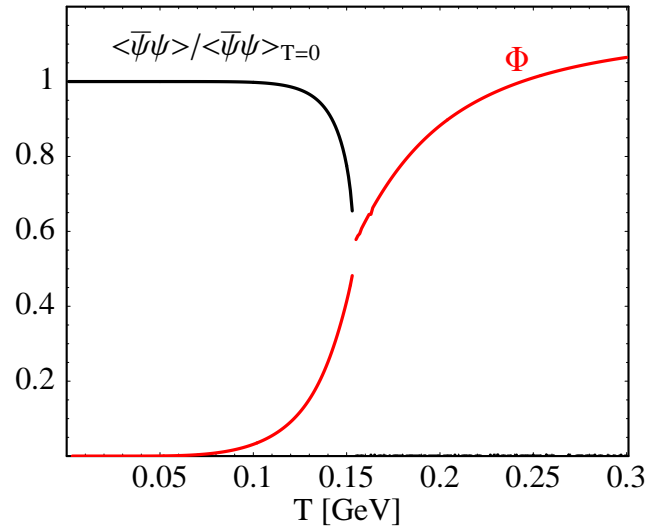
Scaled quark number density as a function of the temperature at different values of the chemical potential, compared to the corresponding lattice data taken from [Z. Fodor *et al.*, PLB 568 \(2003\)](#).

$$N_f = 3 \Leftrightarrow m_s = m_u = 0$$

Phase diagram

Influence of 't Hooft interaction

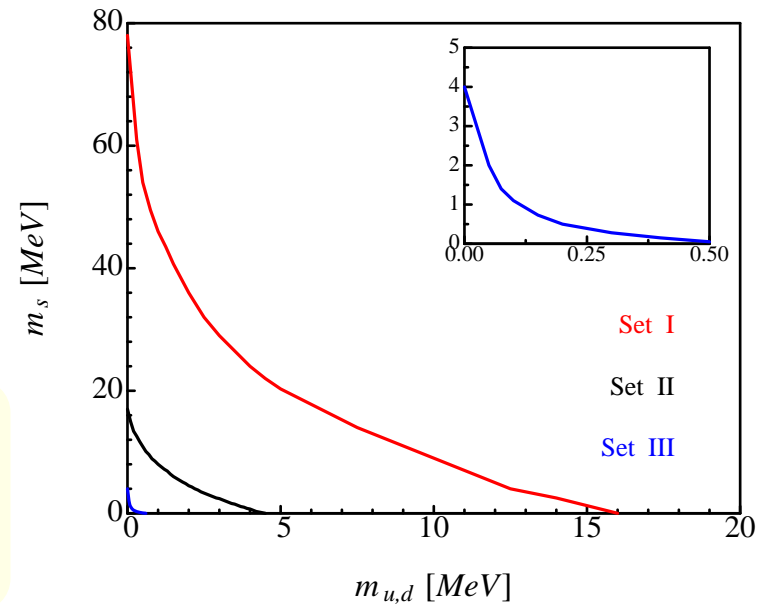
C. R., S. Rößner, M. A. Thaler and W. Weise, in preparation



First order phase transition
for three massless flavours
at $\mu = 0$

	Λ [GeV]	$G\Lambda^2$	$K\Lambda^5$
set I	0.6023	3.072	40.00
set II	0.6023	3.67	24.72
set III	0.6023	4.2426	10.00

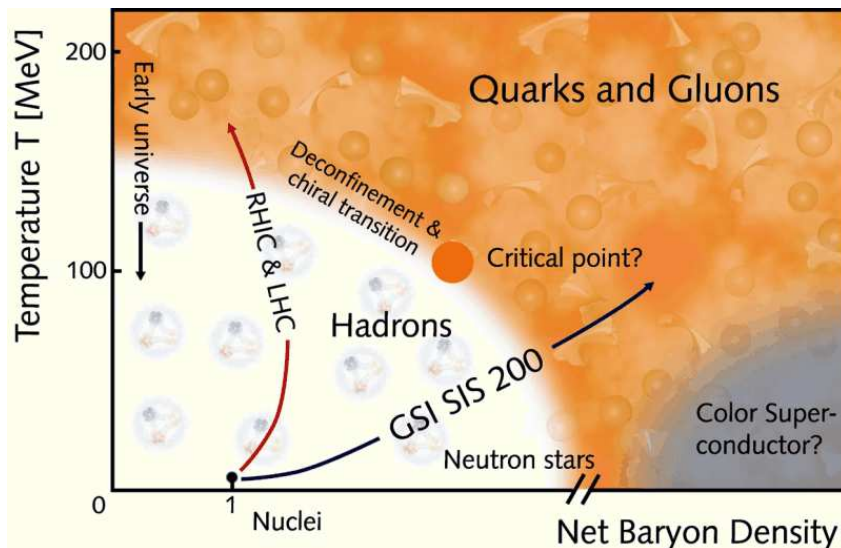
't Hooft six-fermion interaction responsible
for first-order phase transition!



Conclusions

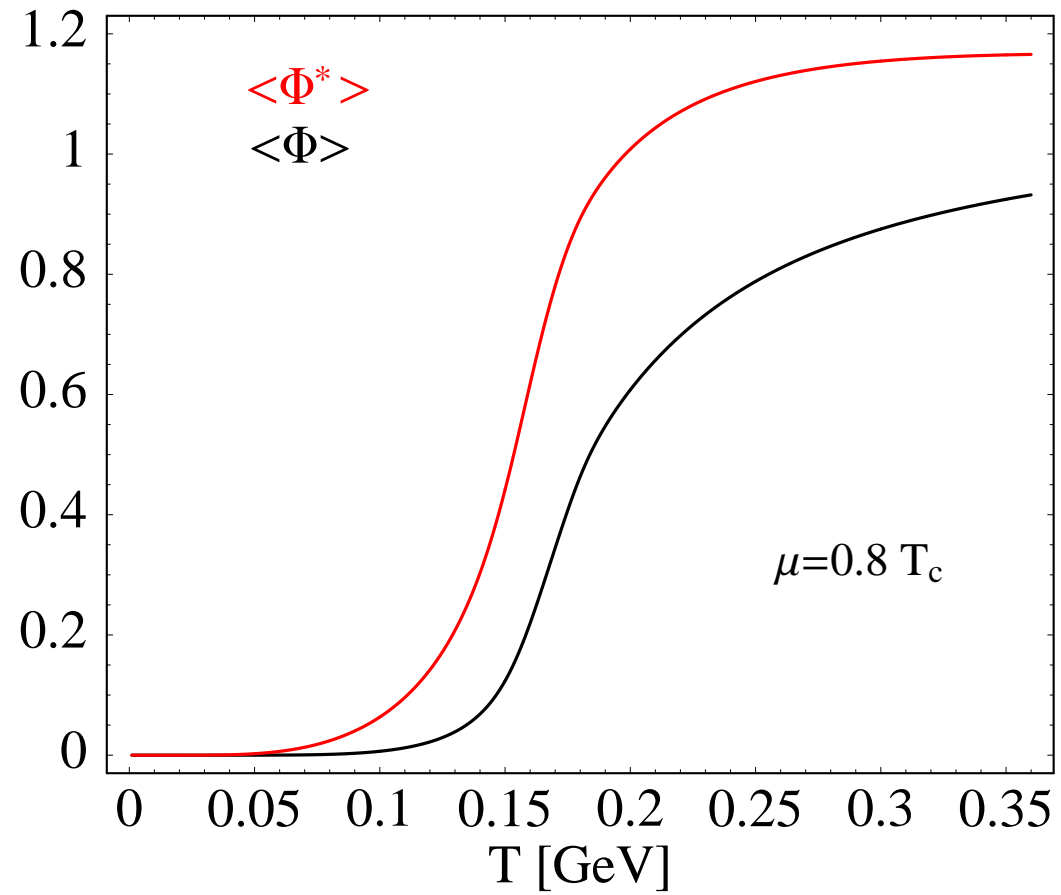
- ❖ The **standard NJL model** fails in reproducing QCD thermodynamics
- ❖ PNJL model as a minimal synthesis of confinement and chiral symmetry breaking
- ❖ A description of QCD thermodynamics with our simple model works very well
- ❖ $N_f = 2$
 - ➔ $\mu = 0$
 - ➔ $\mu \neq 0$
- ❖ $N_f = 2 + 1$
- ❖ $N_f = 3$

Outlook



- ❖ Lattice QCD can reach only **moderate μ**
 - ➔ exploration of **high μ** regions
 - ➔ introduction of diquarks
- ❖ Exploration of the phase diagram and its quark mass dependence for $N_f = 2 + 1$ and $N_f = 3$

Backup slides

Finite μ results

Fixing the parameters

- ❖ In the Polyakov loop potential there are **7** parameters:
 - ➡ $a_0, a_1, a_2, a_3, b_3, b_4, T_0$
 - but **only 3** are free

- ❖ There are 4 constraints:
 - ➡ T_0 is fixed to **270 MeV**, the known critical temperature for the pure gauge system
 - ➡ The Polyakov loop must tend to **1** as $T \rightarrow \infty$
 - ➡ $p(T) = -V(\Phi(T), T)$ tends to the ideal gas limit as $T \rightarrow \infty$
 - ➡ At $T = T_0$ the absolute minimum of $V(\Phi, T)$ must jump from $\Phi = 0$ to a **finite Φ**

- ❖ The remaining three parameters are fixed to reproduce the pure gauge lattice data

Parameter fixing

We have three free parameters in the model: m_0 , Λ , G . They are fixed by:

- ❖ The pion decay constant f_π is evaluated in the NJL model through the following relation:

$$f_\pi^2 = 4m^2 I_\Lambda^{(1)}(m) \quad \text{where} \quad I_\Lambda^{(1)}(m) = -iN_c \int \frac{d^4p}{(2\pi)^4} \frac{\theta(\Lambda^2 - \vec{p}^2)}{(p^2 - m^2 + i\epsilon)^2}.$$

The empirical value is $f_\pi = 92.4 \text{ MeV}$.

- ❖ The quark condensate becomes

$$\langle \bar{\psi}_u \psi_u \rangle = -4m I_\Lambda^{(0)}(m) \quad \text{with} \quad I_\Lambda^{(0)}(m) = iN_c \int \frac{d^4p}{(2\pi)^4} \frac{\theta(\Lambda^2 - \vec{p}^2)}{p^2 - m^2 + i\epsilon}.$$

Its “empirical” value derived from QCD sum rules is

$$\langle \bar{\psi}_u \psi_u \rangle^{1/3} \simeq \langle \bar{\psi}_d \psi_d \rangle^{1/3} = -(240 \pm 20) \text{ MeV}.$$

- ❖ The current quark mass m_0 is fixed from the Gell-Mann, Oakes, Renner (GMOR) relation:

$$m_\pi^2 = \frac{-m_0 \langle \bar{\psi} \psi \rangle}{f_\pi^2}.$$

Thermodynamic potential

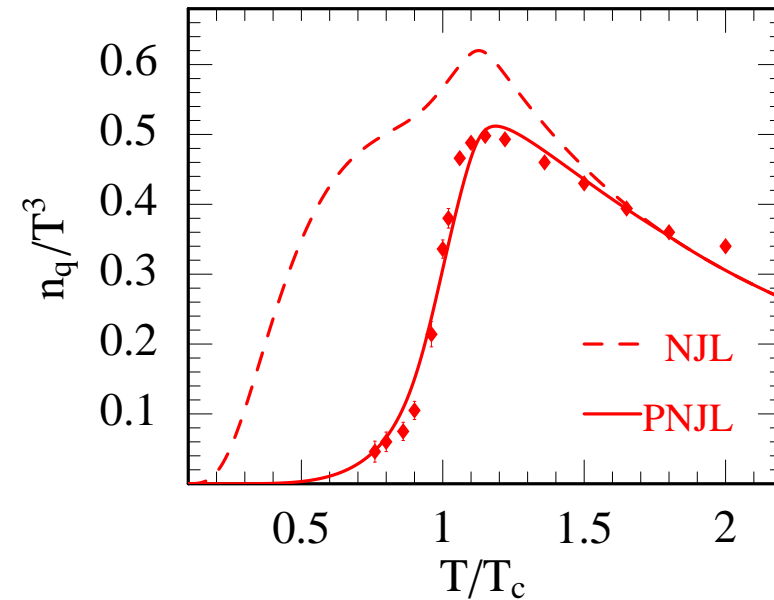
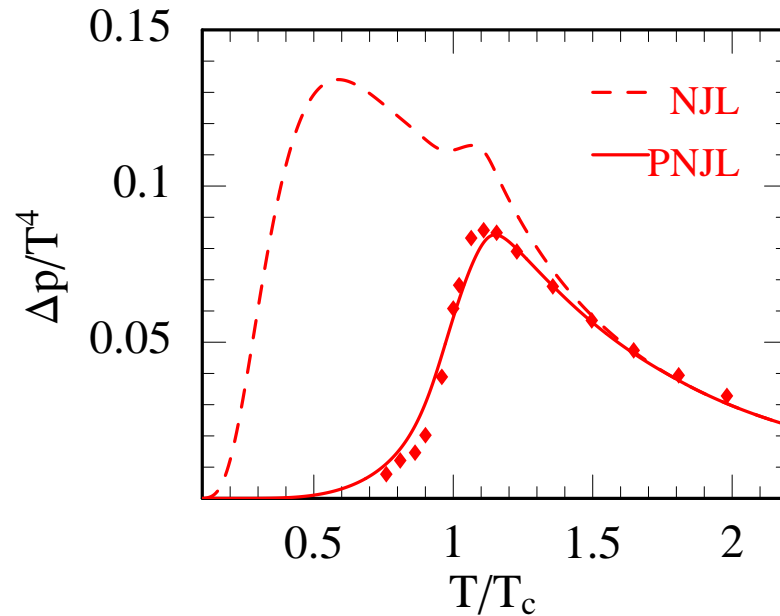
The final form of the thermodynamic potential is

$$\Omega(T, \mu, \sigma, \Phi, \Phi^*) = V(\Phi, \Phi^*, T) + \frac{\sigma^2}{2G} - 2N_c N_f \int \frac{d^3p}{(2\pi)^3} \left\{ E_p + \frac{T}{3} \left[\ln \left[1 + 3 \left(\Phi + \Phi^* e^{-(E_p - \mu)/T} \right) e^{-(E_p - \mu)/T} + e^{-3(E_p - \mu)/T} \right] + \ln \left[1 + 3 \left(\Phi^* + \Phi e^{-(E_p + \mu)/T} \right) e^{-(E_p + \mu)/T} + e^{-3(E_p + \mu)/T} \right] \right] \right\}$$

with $E_p = \sqrt{p^2 + m^2} = \sqrt{p^2 + (m_0 - \sigma)^2}$.

Field equations:

$$\frac{\partial \Omega(T, \mu, \sigma, \Phi, \Phi^*)}{\partial \sigma} = 0, \quad \frac{\partial \Omega(T, \mu, \sigma, \Phi, \Phi^*)}{\partial \Phi} = 0, \quad \frac{\partial \Omega(T, \mu, \sigma, \Phi, \Phi^*)}{\partial \Phi^*} = 0$$

Finite μ results in the standard NJL model

- ❖ Comparison between PNJL and standard NJL model results at finite μ
- ❖ NJL model alone fails in reproducing lattice data
- ❖ Incorporation of confinement **crucial** to reproduce lattice results

$$N_f = 2$$

Parameters

Λ [GeV]	0.651
G [GeV ⁻²]	10.078
m_0 [MeV]	5.5

Physical quantities

f_π [MeV]	92.4
$ \langle \bar{\psi}\psi \rangle ^{1/3}$ [MeV]	247
m_π [MeV]	139.3

$$N_f = 2 + 1$$

Parameters

Λ [GeV]	0.6023
$G\Lambda^2$	3.67
$K\Lambda^5$	24.72
$m_{0u,d}$ [MeV]	5.5
m_{0s} [MeV]	140.7

Physical quantities

f_π [MeV]	92.4
$ \langle \bar{\psi}\psi \rangle_{u,d} ^{1/3}$ [MeV]	241.9
$ \langle \bar{\psi}\psi \rangle_s ^{1/3}$ [MeV]	257.7
m_π [MeV]	139.3
m_K [MeV]	497.7