Rethinking the QCD collisional energy loss

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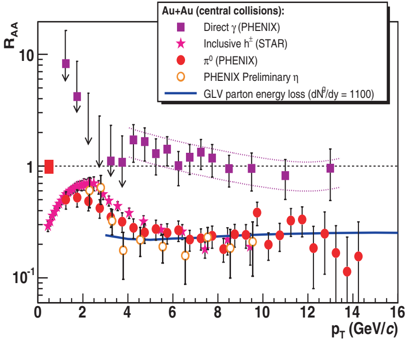
1. Bjorken's formula
2. What is the relevant coupling?
3. dE_{col}^{QCD}/dx

Why could it be interesting?

piece of evidence for **QGP at RHIC**

- nuclear modification factor, hadrons vs. photons



⇒ jet quenching ⇒ **energy loss**

- final state interactions
- radiative loss
  [BDMPS], [GLV], [W], [SW], . . .

expected	'seen'
$\hat{q} \sim 2\mathrm{GeV}^2/\mathrm{fm}$	$\hat{q} \sim 10\mathrm{GeV}^2/\mathrm{fm}$
$\frac{dN_{qg}}{dy} \sim 1000$	$\frac{dN_{qg}}{dy} \sim 2000$

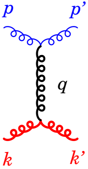
'there must be something else ...'

- **collisional loss**
  [Bjorken], [Braaten, Thoma], . . .
  early estimates $\sim 0.3\mathrm{GeV/fm}$
  now few people say it could be larger

Bjorken's formula

mean energy loss of jet j with energy E in (thermal) medium of scatterers s

$$\frac{dE_j}{dx} = \sum_s \int_{k^3} \rho_s(k)\, \Phi \int dt\, \frac{d\sigma_{js}}{dt}\, \Delta E\,, \quad \Delta E = E - E'$$



- t-channel dominates $\quad \dfrac{d\sigma_{js}}{dt} = 2\pi\, C_{js}\, \dfrac{\alpha^2}{t^2}$
- hard jets ($E, E' \gg k \sim T$): $\quad \begin{aligned} -t &= q^2 = 2(1-\cos\theta)k\Delta E \\ \Phi &= 1 - \cos\theta \end{aligned}$

Bjorken evaluates

$$\Phi \int dt\, \frac{d\sigma_{js}}{dt}\, \Delta E = \frac{2\pi\, C_{js}\alpha^2}{-2k} \int_{t_1}^{t_2} \frac{dt}{t} = \frac{\pi\, C_{js}\alpha^2}{k}\, \ln\frac{t_1}{t_2}$$

Bjorken's formula

cut-offs in t-integral … 'physically intuitive' …

- soft: exchanged gluon is Debye screened, $-t_2 = \mu^2 \sim m_D^2 \sim \alpha T^2$
- hard: kinematic constraint $\Delta E_{max} \sim E/2$, $-t_1 = (1-\cos\theta)kE$

$$\Rightarrow \quad \frac{dE_j^B}{dx} = \pi\alpha^2 \sum_s C_{js} \int_{k^3} \frac{\rho_s}{k} \ln \frac{(1-\cos\theta)kE}{\mu^2}$$

Bjorken 'calculates' $\ln(\%) \to \ln(2\langle k\rangle E/\mu^2)$ with $\langle k\rangle \to 2T$, arriving at

$$\boxed{\frac{dE_{q,g}^B}{dx} = \left(\tfrac{2}{3}\right)^{\pm 1} 2\pi \left(1+\tfrac{1}{6}\, n_f\right) T^2\, \alpha^2 \ln \frac{4TE}{\mu^2}}$$

[Bjorken '82]

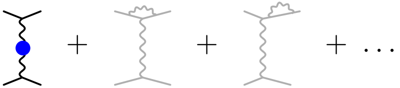
relativistic adaption of (QED) Bethe-Bloch formula

NB: HTL calculations [Thoma et al.] to fix cut-offs: $\mu^\star = m_D$

What is α?

Loop corrections (massless QED)

beyond tree level



(cancellation due to $Z_1 = Z_2$)

1. $T = 0$

bare coupling

eff. matrix element $\mathcal{M} = \dfrac{\alpha}{P^2 - \Pi_{\rm vac}}$

unrenormalized self-energy

$$\Pi_{\rm vac}(P^2) = 4\pi\beta_0\alpha \left[\epsilon^{-1} - \ln \frac{-P^2}{\bar\mu^2}\right] P^2$$

coupling renormalization

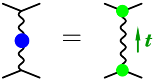
- measure at $P^2 = t_e$: $\mathcal{M}(t_e) = \frac{1}{t_e}\,\frac{\alpha}{1-4\pi\beta_0\alpha[\epsilon^{-1}-\ln(-t_e/\bar\mu^2)]} \equiv \boxed{\frac{\alpha(t_e)}{t_e}}$

  observable!

  $\Rightarrow \alpha^{-1}(t_e) = \alpha^{-1} - 4\pi\beta_0\left[\epsilon^{-1} - \ln(-t_e/\bar\mu^2)\right]$ ‘running coupling’

- predict at t: $\alpha^{-1}(t) = \alpha^{-1}(t_e) + 4\pi\beta_0 \ln(t/t_e)$ (**RG**), or

$$\boxed{\alpha(t) = \left[4\pi\beta_0 \ln(|t|/\Lambda^2)\right]^{-1}}$$



Loop corrections (massless QED)

2. $T > 0$

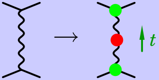
additional contribution to selfenergy components ($i = t, l$)

$$\Pi^i(p_0, p) = 4\pi\beta_0\alpha\left[\left(\epsilon^{-1} - \ln(-P^2/\bar{\mu}^2)\right)P^2 + f^i(p_0, p)\right]$$

rewrite ('renormalize') in-medium scattering matrix

$$\frac{\alpha}{P^2 - \Pi^i} = \frac{P^{-2}}{\alpha^{-1} - 4\pi\beta_0[\epsilon^{-1} - \ln(-P^2/\bar{\mu}^2) + f^i/P^2]}$$

$$= \frac{P^{-2}}{\alpha^{-1}(P^2) - 4\pi\beta_0 f^i/P^2} = \frac{\alpha(P^2)}{P^2 - 4\pi\beta_0\alpha(P^2)f^i} = \frac{\alpha(P^2)}{P^2 - \Pi^i_T|_{\alpha(P^2)}}$$



- effective IR cut-off $\mu^2 \sim \Pi_T(0^-) \sim m_D^2$
- running coupling **α(t) at virtuality**

Running coupling

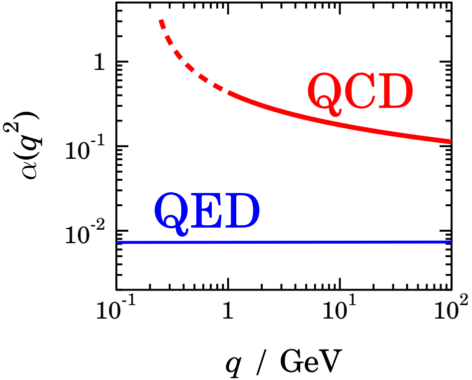
1-loop approximation $\alpha(t) = \dfrac{1}{4\pi\beta_0 \ln(-t/\Lambda^2)}$

- QED

$$\begin{aligned}\beta_0 &= -\tfrac{4}{3}/(4\pi)^2 \\ \Lambda &\sim 10^{xx}\,\mathrm{GeV}\end{aligned}$$

- QCD

$$\begin{aligned}\beta_0 &= (11-\tfrac{2}{3}n_f)/(4\pi)^2 \\ \Lambda &\approx 0.2\,\mathrm{GeV}\end{aligned}$$



QCD: **running coupling crucial** at *probable* scales

dE_{col}^{QCD}/dx

- reconsider t-integration in energy loss

$$\Phi \int_{t_1}^{t_2} dt \, \frac{d\sigma_{js}}{dt} \, \Delta E = -\frac{\pi C_{js}/k}{(4\pi\beta_0)^2} \int_{t_1}^{t_2} \frac{dt}{t \ln^2(|t|/\Lambda^2)} = \frac{\pi C_{js}/k}{(4\pi\beta_0)^2} \, \frac{1}{\ln(|t|/\Lambda^2)} \Bigg|_{t_1}^{t_2}$$

$$= \frac{\pi C_{js}/k}{4\pi\beta_0} \left[\alpha(\mu^2) - \alpha(t_1)\right] \quad \text{instead of} \quad \frac{\pi C_{js}}{k} \, \alpha^2 \ln \frac{t_1}{\mu^2}$$

for hard jets, due to $|t_1| \sim TE \gg \mu^2 \sim m_D^2 \sim \alpha T^2$, independent of E

- QCD collisional energy loss (with running coupling)

$$\frac{dE_j}{dx} = \pi \frac{\alpha(\mu^2)}{4\pi\beta_0} \sum_s C_{js} \int_{k^3} \frac{\rho_s(k)}{k} \;\rightarrow\; \left(\tfrac{2}{3}\right)^{\pm 1} 2\pi \left(1 + \tfrac{1}{6}\, n_f\right) T^2 \, \frac{\alpha(\mu^2)}{4\pi\beta_0}$$

  - parametric enhancement: order α instead of α^2
  - scale of running coupling is $\mu \sim m_D \sim \sqrt{\alpha}\,T$ instead of $\mathcal{O}(T)$
  - independent of jet energy instead of $\sim \ln E$

Debye mass

Debye mass describes screening of static (color) electric fields

- defined as pole of longitudinal propagator

$$m_D^2 = \Pi^l(0,p)\Big|_{p^2=-m_D^2}$$

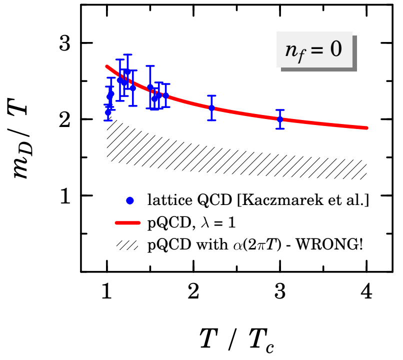
renormalize, express pole in terms of running coupling

$$P^2 = \#\alpha(P^2)\, f(p_0,p)$$

- **implicit formula** for Debye mass

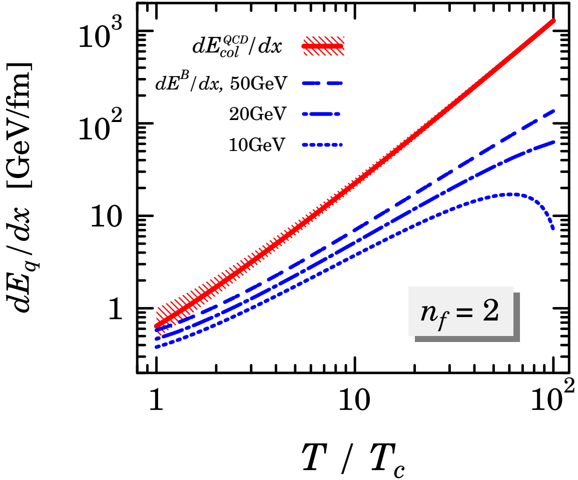
$$\boxed{m_D^2 = \tfrac{1}{3} N_c \left(1+\tfrac{1}{6} n_f\right) 4\pi\alpha(m_D^2)\, T^2}$$

[AP, hep-ph/0601119]



$$\lambda_{fit}^{(0)} \approx \frac{1}{1.14} = \frac{\Lambda^{(0)}}{T_c^{(0)}}\Bigg|_{lattice}$$

Extrapolations, estimates



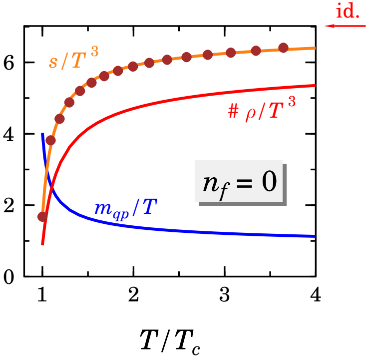
- $T_c = 160$ MeV
- $\Lambda = 205$ MeV (consistent with lattice results for *both* $m_D(T)$ and $V(r)$ at $T = 0$)
- $\mu^2 = [\frac{1}{2}, 2]m_D^2$
- dE^B/dx calc. with $\alpha(2\pi T)$, and $\mu = m_D$

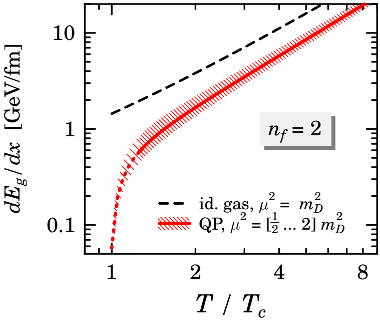
sizable enhancement, *even* near T_c, compared to previous estimates

Extrapolations, estimates

bulk properties of sQGP described by **quasiparticles** with mass $m_{qp} \sim \sqrt{\alpha}\,T$
[AP, Kämpfer, Soff]

$$\frac{dE}{dx} \sim \int_{k^3} \rho(k)\,\Phi \int dt\,\frac{d\sigma}{dt}\,\Delta E$$





near T_c: large coupling → reduced phase space density → **small** dE/dx

Conclusions

- leading order QCD collisional energy loss

$$\boxed{\frac{dE_{col}^{QCD}}{dx} \sim \alpha(m_D)\,T^2}$$

  - independent of jet energy
  - non-Abelian enhancement compared to QED analog

- estimates for HIC's (based on parameters consistent with lattice QCD)
  - for $T \lesssim 1.5\,T_c$: strongly interacting, yet **transparent QGP**
    $\Rightarrow$ no quenching at SPS
  - for $T \gtrsim 1.5\,T_c$: considerably **larger than previous estimates**
    $\Rightarrow$ strong collisional quenching at RHIC/LHC