Formation time scaling and hadronization in cold nuclear matter

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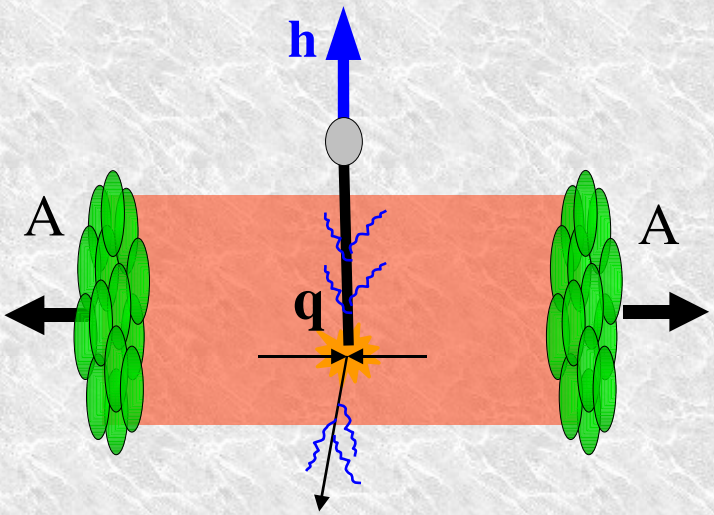
Can we distinguish
partonic energy loss from (pre)hadron absorption?

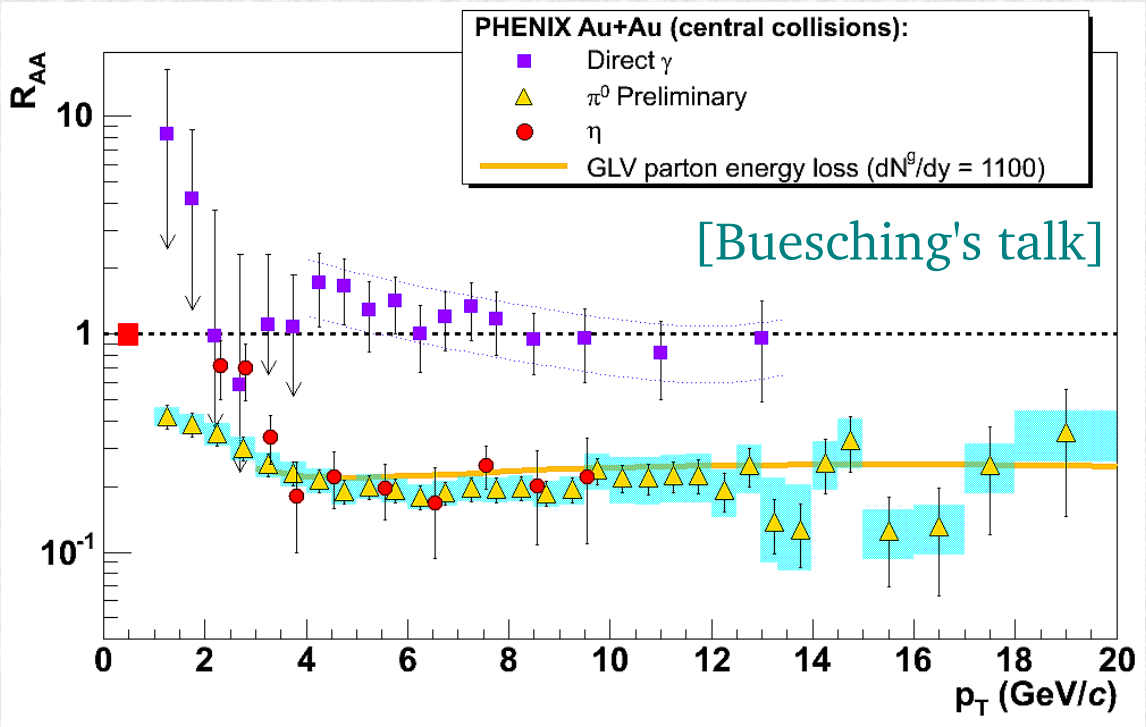
- the short answer: **YES!**
- from experimental data
- with minimum model dependency

based on A.A., nucl-th/0604041

Radiative energy loss: success!



$$R_{AA} = \frac{(dN^h/d^2p_T)_{A+A}}{N_{coll}(b)\,(dN^h/d^2p_T)_{p+p}}$$



- Used to extract initial gluon density

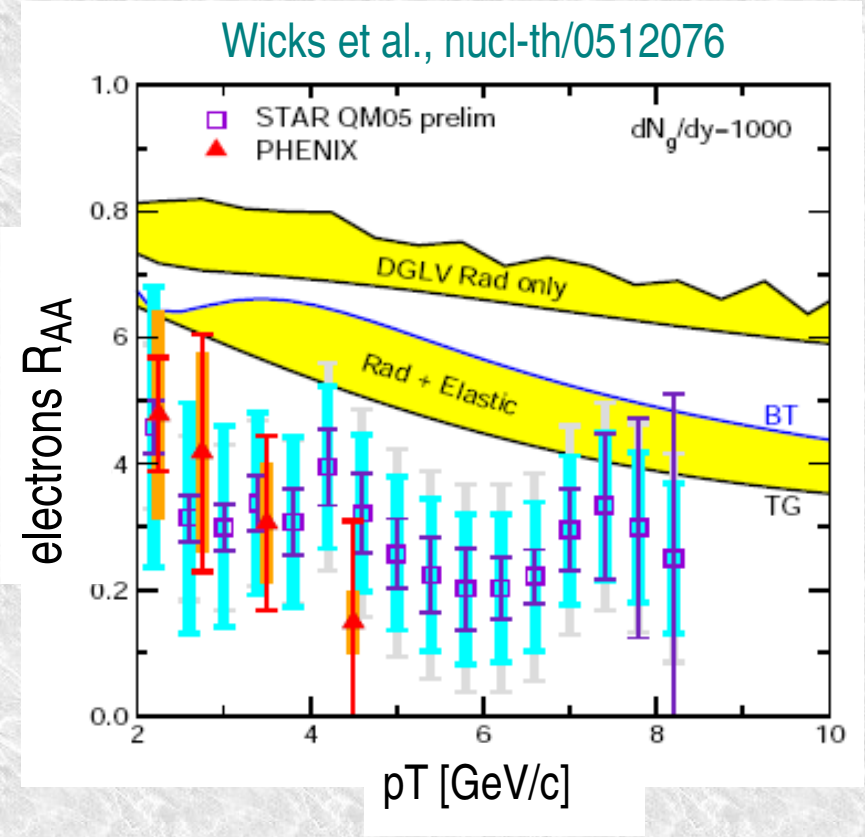
$$dN^g/dy \approx 1000$$

- Can be converted to initial state energy density:

$$\varepsilon \sim 14 - 20\ \mathrm{GeV/fm^3} \gg \varepsilon_c$$

Success?

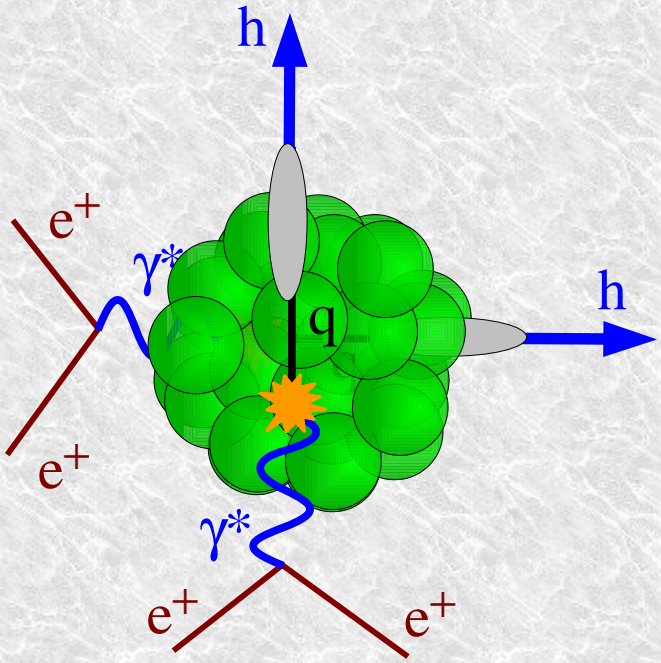
- Heavy flavour puzzle at RHIC [QM2005, STAR, PHENIX, Djordjevic, Armesto]
  - single non-photonic e^- as much suppressed as π
  - ***theory gives half of the observed suppression!***

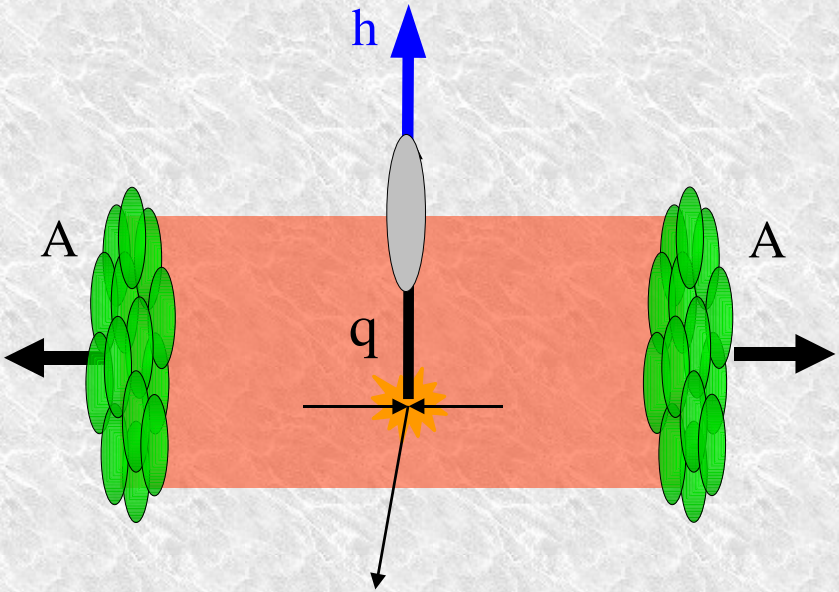


- Additional mechanisms
  - elastic energy loss?
    with running α_s? [Peshier's talk]
  - something else?

- A big untested assumption:
  - partons live long enough to traverse the whole medium
  - let's test it! ...in nuclear Deep Inelastic Scattering (**nDIS**)

nDIS vs. A+A collisions





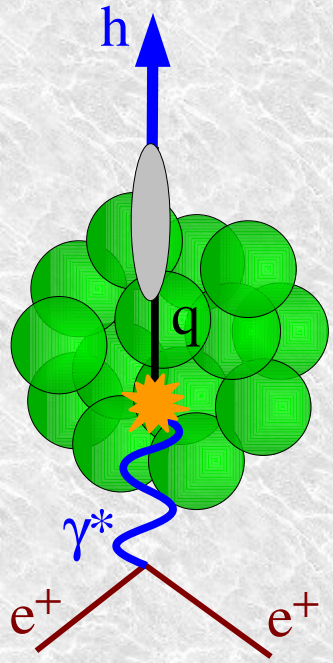
- nDIS is a clean environment for
  - (1) **space-time evolution of hadronization**
    - nucleons as micro-detectors
    - medium rather well known
  - (2) **Cold nuclear matter effects**
    - quark energy loss
    - nuclear modifications of FF

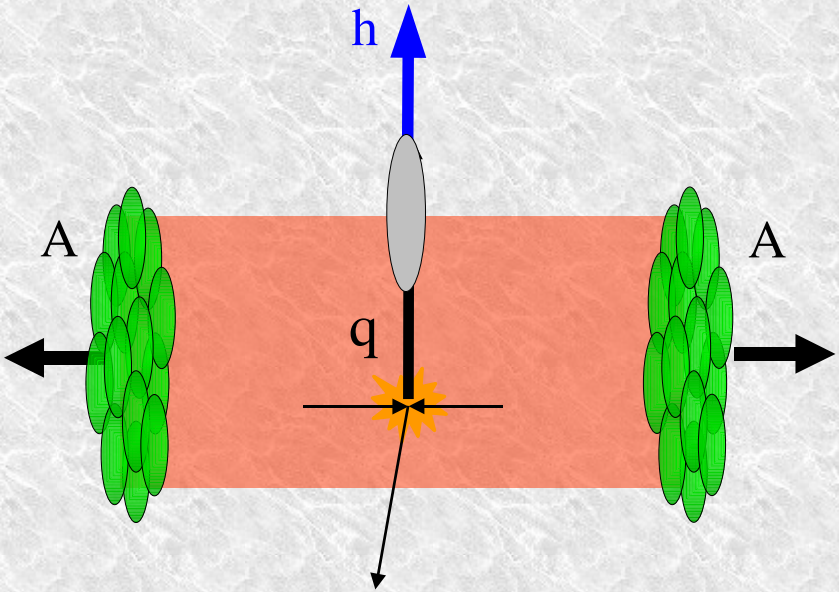
Jet-quenching in A+A

⇓

properties of hot nuclear matter

Similarities and differences





$E_q = \nu = E_e - E_{e'} \approx$ 2-25 GeV

at HERMES

$E_h = z\,\nu \approx$ **2 - 20 GeV**

$E_q = p_T / z$

$E_h = p_T \approx$ **2 - 20 GeV**

★ HERMES kinematics is relevant to RHIC mid-rapidity

...but beware the virtuality...

$Q^2 = -q^2$ is measured

$Q^2 \equiv E_q^2 \propto (p_T/z)^2$

...and the rapidity...

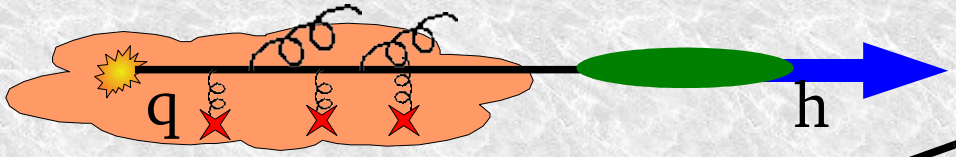
always forward rapidity

rapidity can change

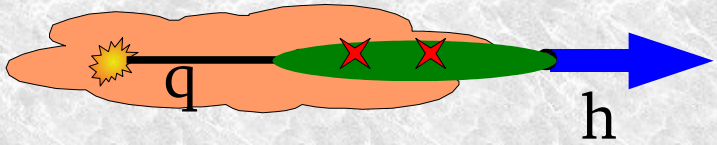
Hadron attenuation in nDIS

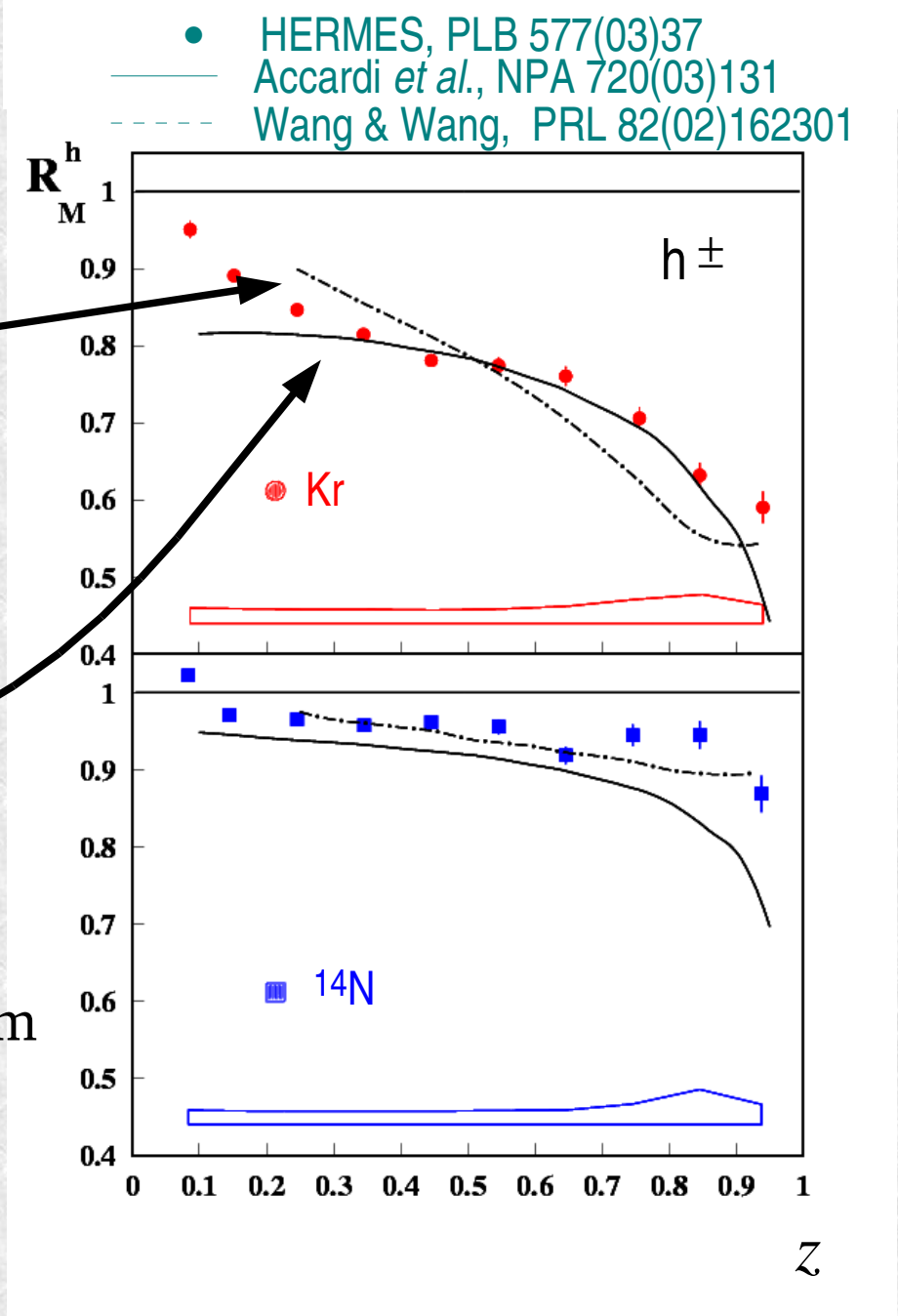
$$R_M^h(z) = \frac{\frac{1}{N_A^{DIS}} \frac{dN_A^h(z)}{dz}}{\frac{1}{N_D^{DIS}} \frac{dN_D^h(z)}{dz}}$$

- **Energy loss (gluon bremsstrahlung)**
  [Arleo; Wang *et al.*]
  - hadronization outside the medium
  - gluon radiation off struck quark



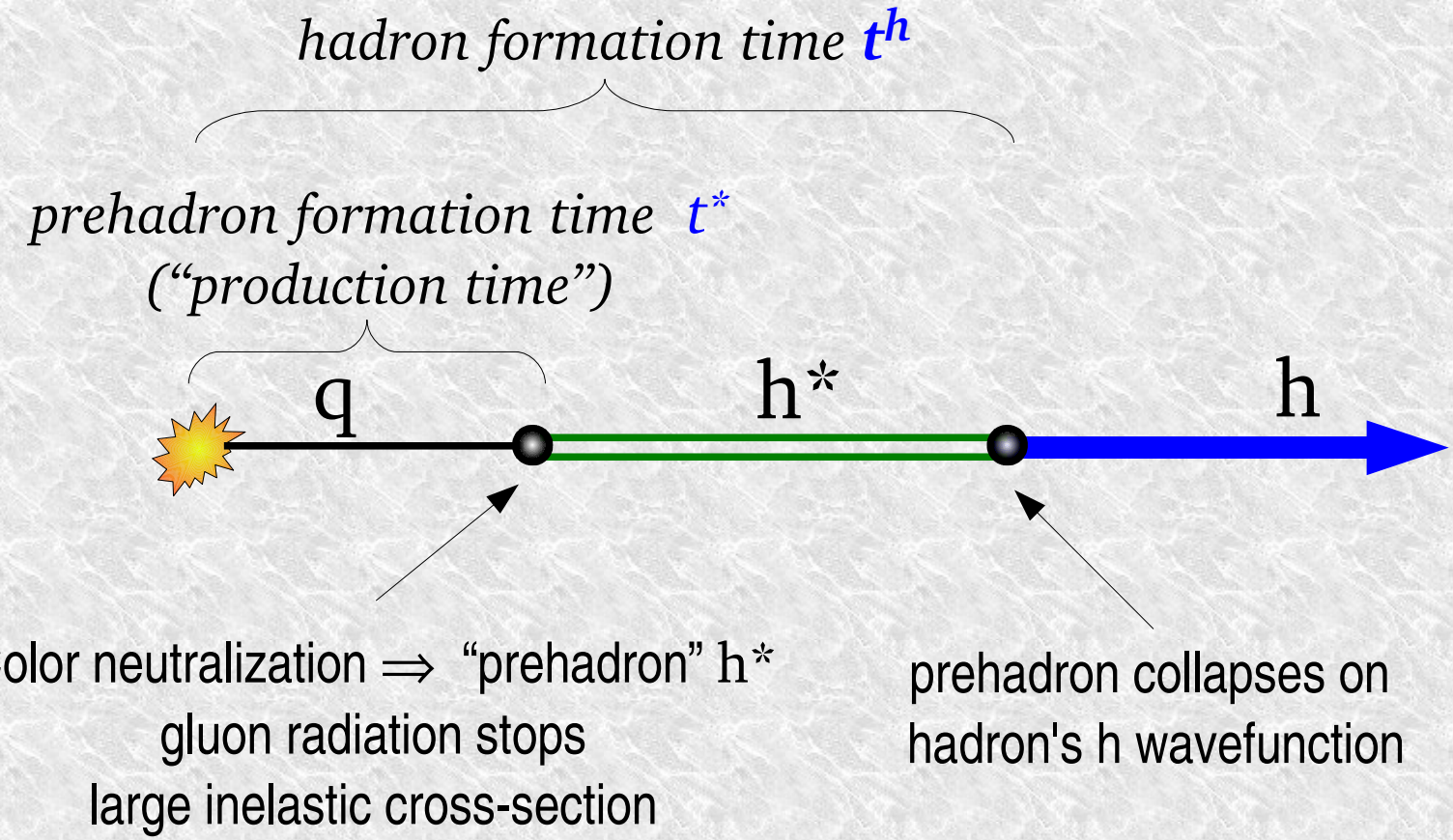
- **Prehadron absorption**
  [Accardi *et al.*;
  Falter *et al.*; Kopeliovich, *et al.*]
  - color neutralization inside the medium
  - prehadron-nucleon scatterings





Prehadron vs. hadron

- Hadronization is non perturbative $\Rightarrow$ (many) models
- General features:



- *NOTE:*
  - It's tricky to rigorously define t^* , t^h: consider them as working tools

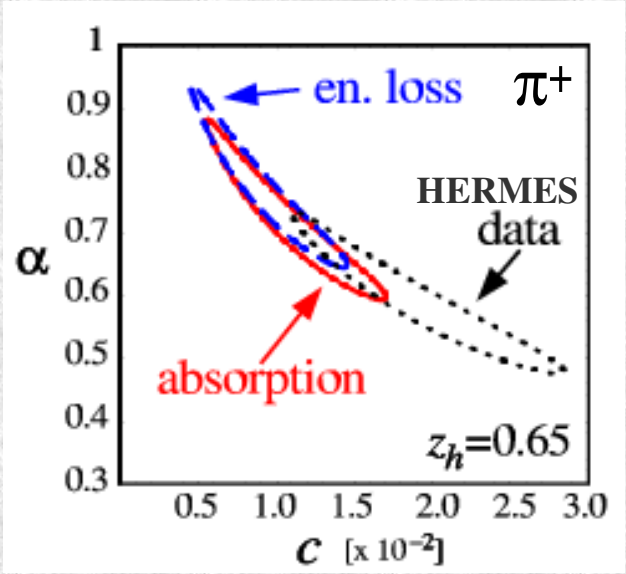
How to tell energy loss from absorption?

- Conventional thinking: the $A^{2/3}$ law
  - Energy loss (LPM effect): $1-R_M \sim \langle\Delta z\rangle \sim L^2 \sim A^{2/3}$
  - Hadron absorption: $1-R_M \sim \langle \text{no. of nucleons seen} \rangle \sim L \sim A^{1/3}$

WRONG!

- $A^{2/3}$ also for absorption models!
  - extra dimensionful scale: prehadron formation length $\langle l^* \rangle$
  - neutralize it ⇒ extra power of A $(R_A/\langle l^* \rangle)^n \sim A^{n/3}$
  - typically $n=1$
- Don't believe it?
  - try fitting $1-R_M = c\,A^{\alpha}$

    [Accardi et al., NPA 761 (05);
    Accardi, Acta Phys. Hung. in press]



Scaling of R_M – basic idea

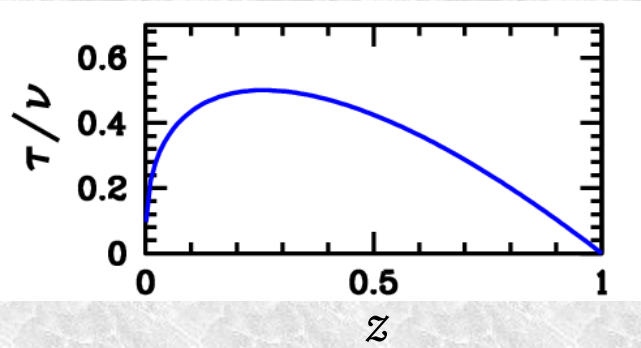
A.A., nucl-th/0604041

- $\mathbf{R_M}$ should scale with $\boldsymbol{\tau = \tau(z,\nu)}$ not with z and ν separately

$$R_M = R_M[\tau(z,\nu)] \quad \text{with} \quad \tau = C\, z^{\lambda} (1-z)\, \nu$$

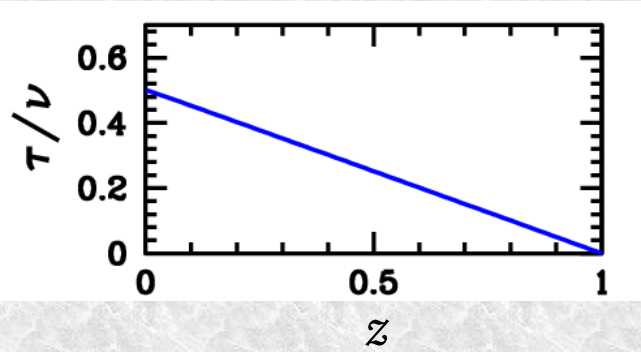
- "Scaling exponent" λ can distinguish absorption and energy-loss

  - **absorption models**: $\lambda > 0$
    [finite formation time]

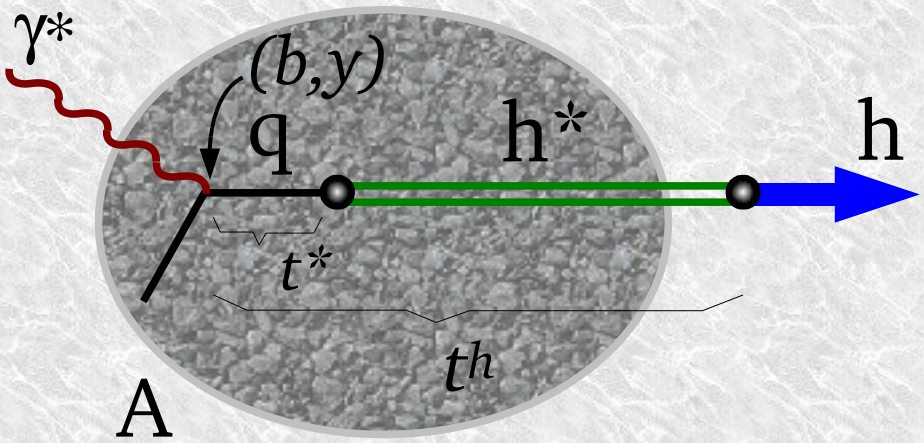


  - **energy loss models**: $\lambda \lesssim 0$
    [from energy conservation:
    $\varepsilon < (1-z_h)\, \nu$]

radiated energy



Scaling of R_M – Absorption models



- R_M depends only on the prehadron's in-medium path $\Leftrightarrow$ $\langle t^* \rangle$

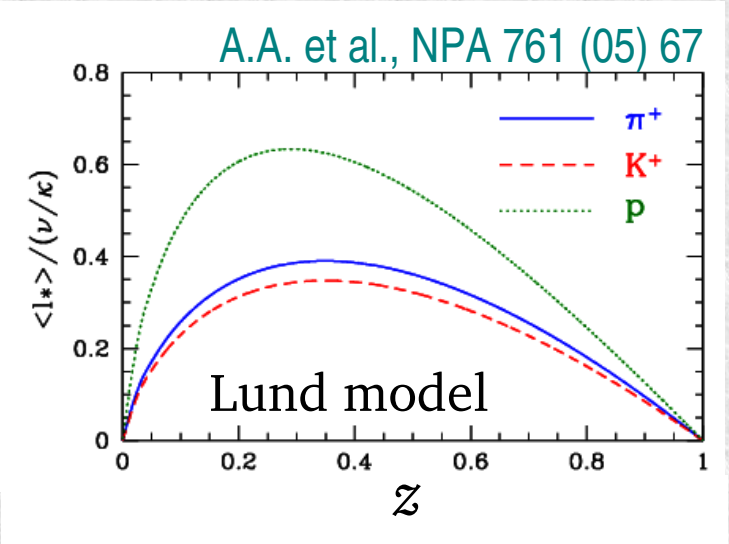
Lorentz boost

$$\langle t^* \rangle = f(z)\,(1-z)\,\frac{z\,\nu}{\kappa} \sim \tau(z,\nu)$$

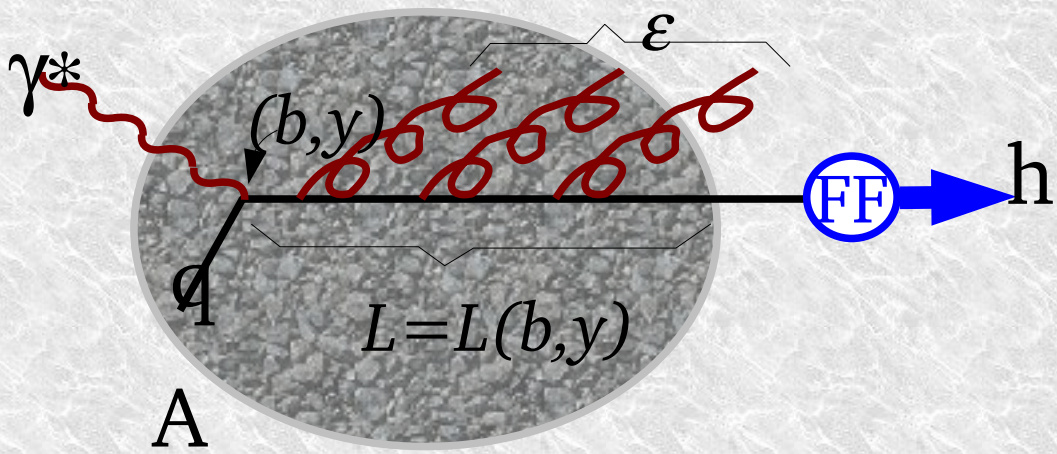
energy conservation

model dep. scale (e.g. string tension)

- scaling with $\lambda > 0$
- for a 10 GeV π: $\langle t^* \rangle \sim 4$ fm $\langle t^h \rangle \sim 10$ fm



Scaling of R_M – Energy loss models



[see Arleo EPJ C30(02)213]

- quark fragments with $E_q - \varepsilon = \nu - \varepsilon \Rightarrow$ modified Fragm. Function

$$\tilde{D}_A(z_h) = \int_0^{(1-z_h)\nu} d\epsilon\, \mathcal{P}(\epsilon)\, \frac{1}{1-\epsilon/\nu} D\left(\frac{z_h}{1-\epsilon/\nu}\right)$$

Quenching Weight = prob. of radiating ε

- define average energy loss:

energy conservation: $\varepsilon < \nu - E_h$

$$\langle \epsilon \rangle = \int_0^{(1-z_h)\nu} d\epsilon\, \epsilon\, \mathcal{P}(\epsilon) = f[(1-z_h)\nu]$$

$\sim \tau(\lambda=0)$

Scaling of R_M – Energy loss models

- Approximate:

$$R_M \approx \frac{1}{1-\langle\epsilon\rangle/\nu} D\left(\frac{z_h}{1-\langle\epsilon\rangle/\nu}\right) \Big/ D(z_h)$$

- Use KKP at $Q^2 = 2\ \mathrm{GeV}^2$ $\quad D(z_h) = C z_h^{\alpha}(1-z_h)^{\beta}$

- Obtain **approximate τ scaling with $\lambda \approx 0$**:

$$R_M \approx \frac{1}{\left(1-\frac{1}{\nu} f[(1-z_h)\nu]\right)^{\alpha+\beta+1}} \left(1-\frac{f[(1-z_h)\nu]}{(1-z_h)\nu}\right)^{\beta}$$

in practice, $\lambda \lesssim 0$

Scaling analysis

A.A., nucl-th/0604041

- HERMES data presented as:
  - z-distributions: $R_M(z)$ with $\langle\nu\rangle = \langle\nu\rangle(z)$
  - ν-distributions: $R_M(\nu)$ with $\langle z\rangle = \langle z\rangle(\nu)$
- Scaling analysis of R_M – remember $\tau = C\, z^{\lambda} (1-z)\, \nu$

1) Fix λ
2) For each z compute $\tau=\tau(z,\langle\nu\rangle)$ and $R_M(\tau) = R_M(z)$
   For each ν compute $\tau=\tau(\langle z\rangle,\nu)$ and $R_M(\tau) = R_M(\nu)$
3) Fit $\phi(\tau)$ to $\{\tau,R_M\}$ obtained above – $\chi^2 = \chi^2(\lambda)$

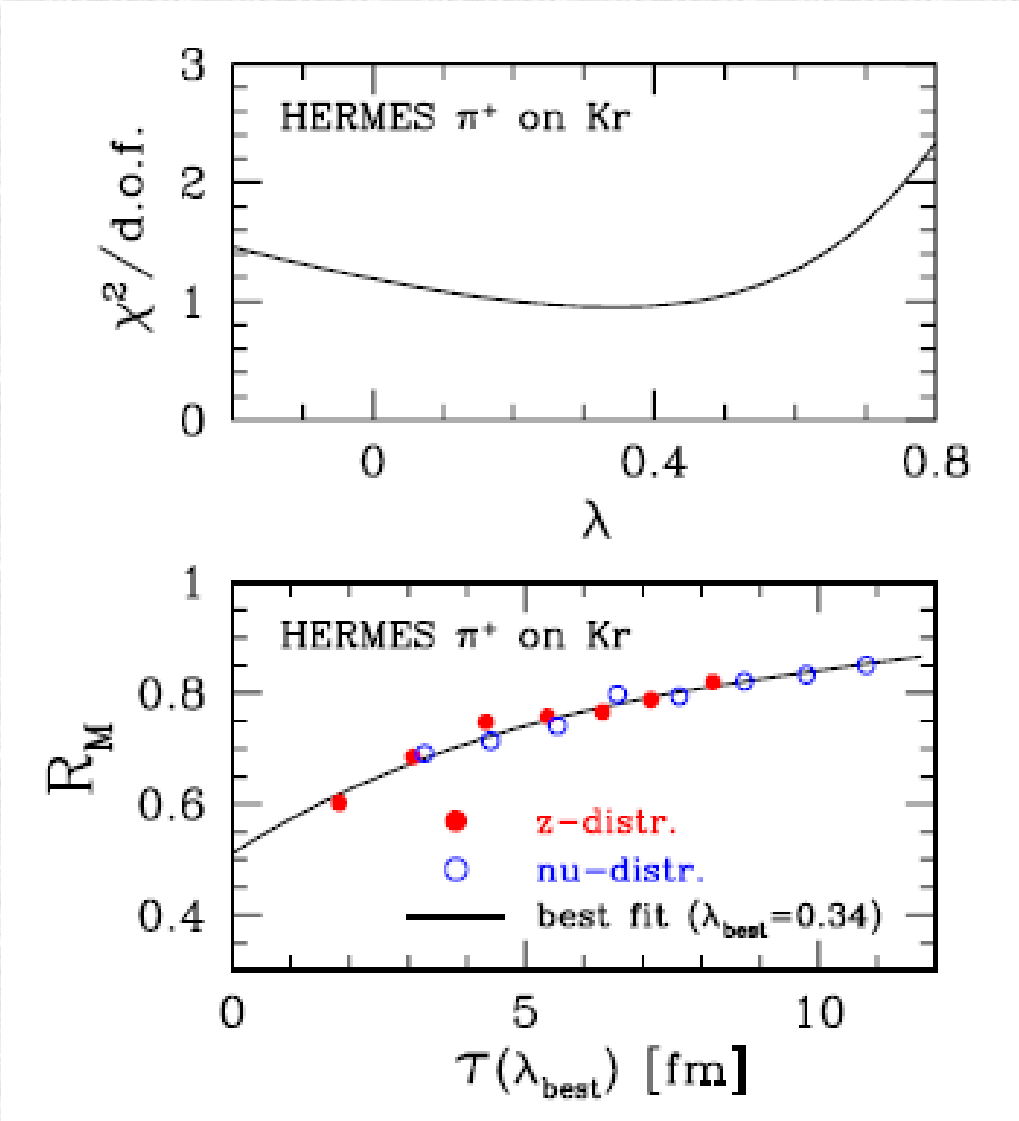
$$\phi(\tau)=a+b\tau+c\tau^2+d\tau^3+e\tau^4$$

(cross-checked by $\phi(\tau)=\dfrac{a+b\tau+c\tau^2}{a+d\tau+e\tau^2}$)

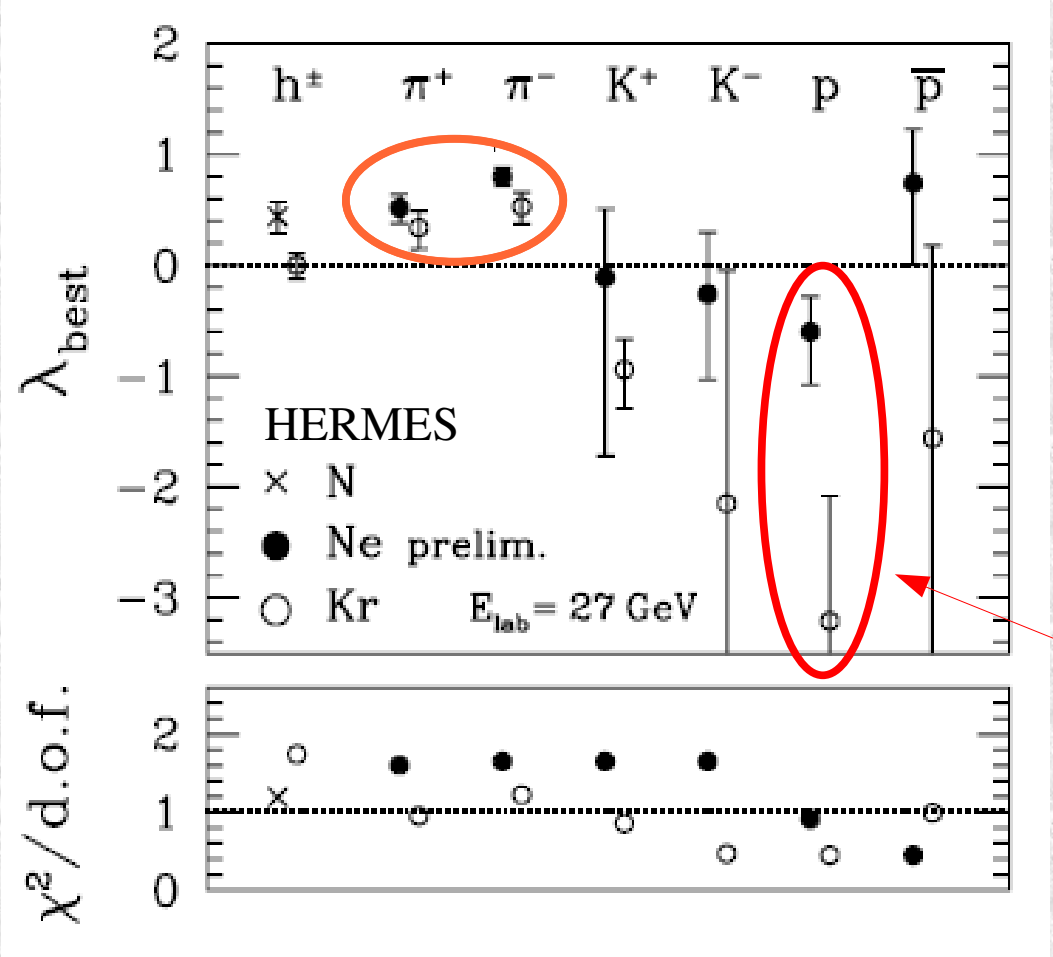
4) Best-fit λ_{best} by χ^2 minimization
5) If $\chi^2(\lambda_{best})/\text{d.o.f.} \lesssim 1 \Rightarrow R_M$ scales with τ, and is charachterized by λ_{best}

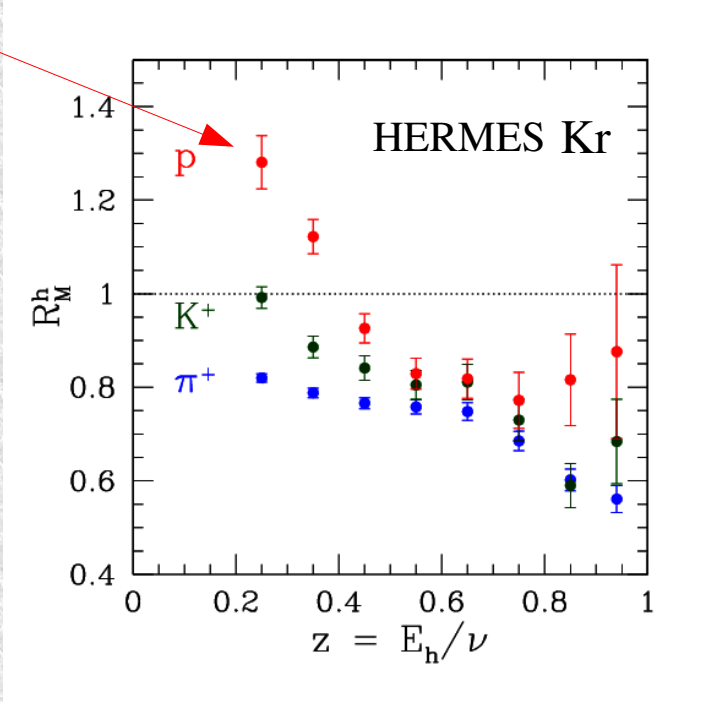
- NOTE: the overall scale of τ cannot be extracted
  (R_M scales, or doesn't scale, independently of the value of the overall constant C)

Scaling analysis - example



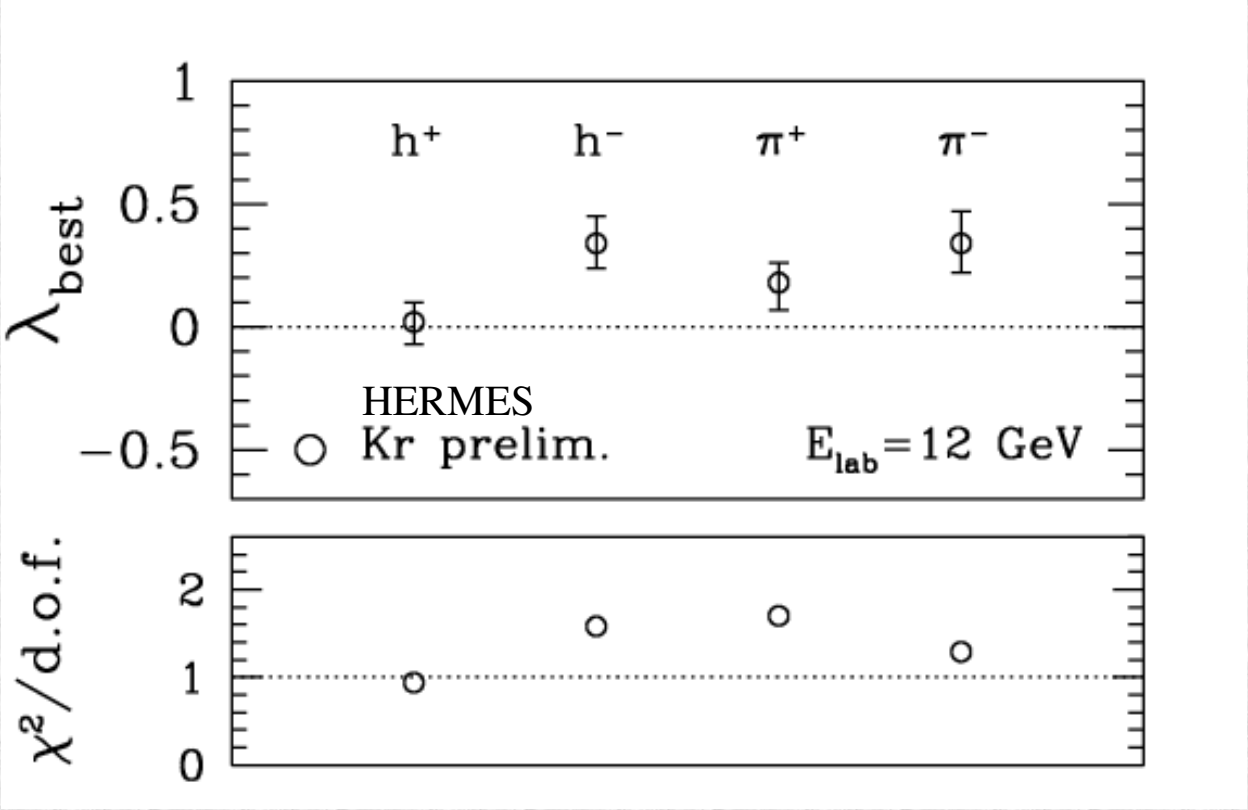
Results – E_{lab} = 27 GeV





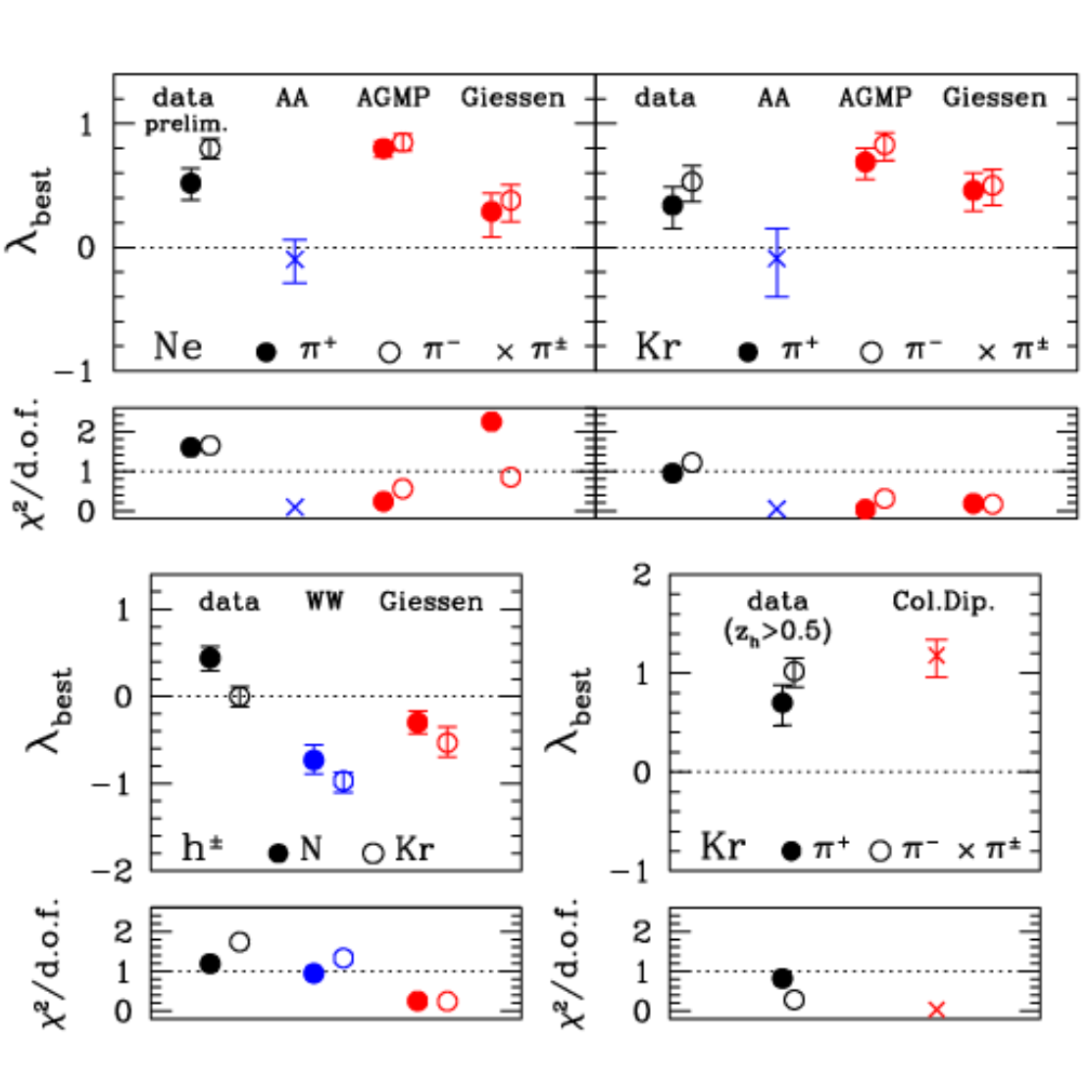
- $\lambda(\pi)>0$: **Formation-time scaling for pions!**
- Why $\lambda_{best}(h^{\pm}) \sim 0$ on Kr?
  - proton anomaly!

Results – E_{lab} = 12 GeV



- pions are still positive! confirms results at 27 GeV
- $\lambda_{best}(h^+) \sim 0$ but $\lambda_{best}(h^-) > 0$
  - proton anomaly hypothesis confirmed!

Data vs. theory



- **Experimental data:**
  - HERMES
- **Energy loss models:**
  - AA = Arleo + realistic geometry (A.A.)
  - WW = Wang & Wang
- **Absorption models:**
  - AGMP = Accardi, Gruenewald, Muccifora, Pirner
  - Giessen = Falter et al.
  - Col.Dip. = Kopeliovich et al.

Conclusions

- Correct interpretation of hadron suppression in A+A **requires** understanding of partonic energy loss vs. prehadronic absorption

- **τ-scaling analysis** of hadron quenching in nDIS:
  - **discriminates energy loss and prehadron absorption**
  - directly from data – with minimal model dependence
  - is powerful (e.g., sensitive to proton anomaly)

- pions and charged hadrons at HERMES have $\lambda_{best} = 0.4 \pm 0.1$

$\Longrightarrow$ **formation-time scaling!**

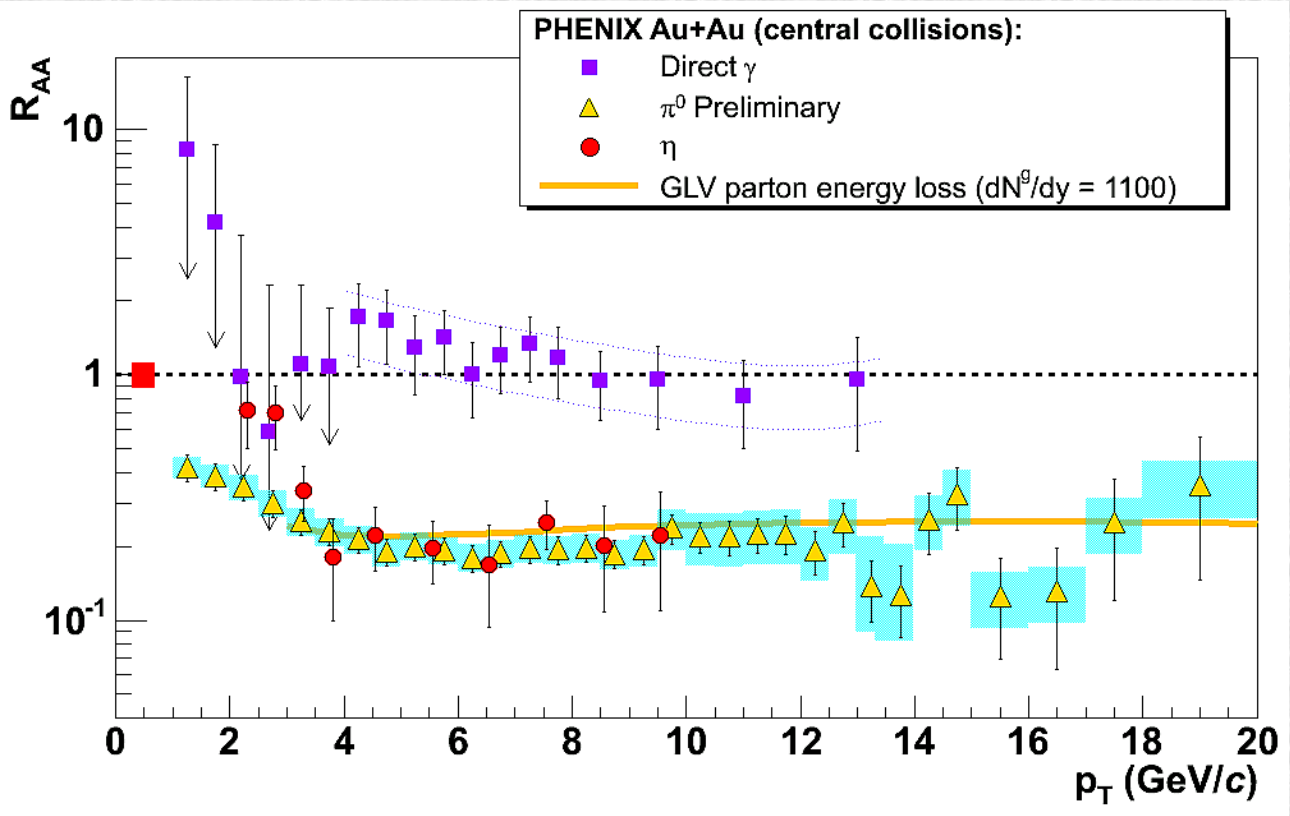
hadronization starts already inside the medium!

  - How much inside? Other observables: p_T-brodening, Cronin effect, ...
  - HERMES @ DESY, CLAS @ JLAB

- A new challenge to the energy loss paradigm in A+A?

Outstanding questions

- Is there a fundamental difference between hadronization in nDIS (forward rapidity) and hadronization in p+p, A+A (midrapidity)?
- Why is η as much suppressed as π in Au+Au collision?



- Enough phenomenology in nDIS: can we address the space-time evolution of hadronization at a more fundamental level? in pQCD?

Backup slides

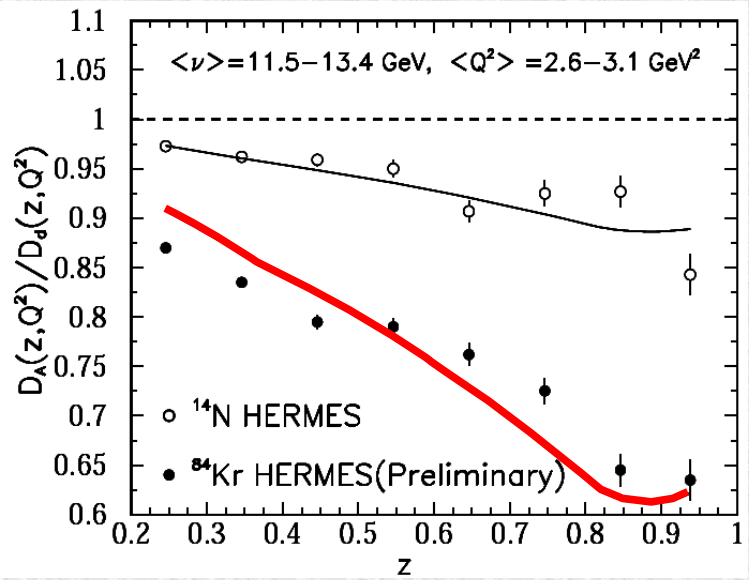
Hadron formation time

Formation time estimates 1

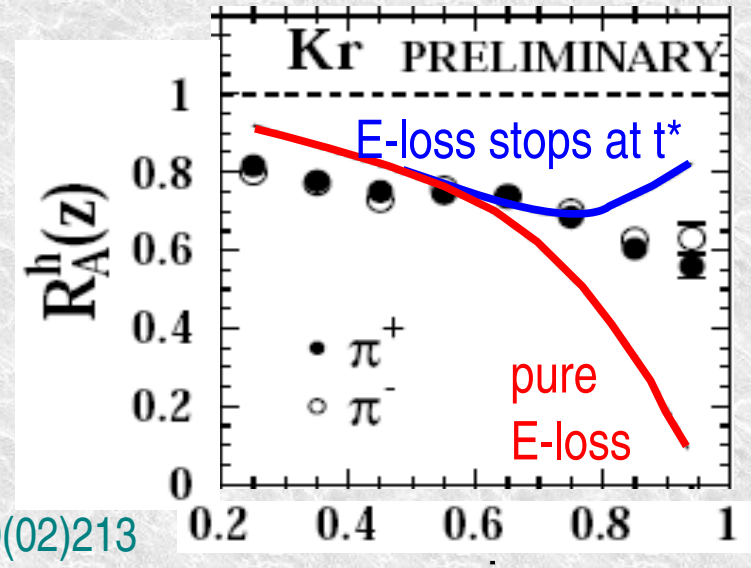
- Hadron formation time = time for partons to build up a color field and develop hadron wave function
  - hadron's rest frame: $\tau^h \sim R_h$
  - lab frame: $t^h \sim R_h \frac{E_h}{m_h}$ (in DIS $t^h \sim R_h \frac{z\nu}{m_h}$)

boost factor

- For a 10 GeV pion at Hermes $t^h \sim 45$ fm $>> R_A$
- This is used in energy loss models to justify the assumptions, but **neglects interactions of the forming color field with the medium**



Wang et al.
PRL 82(02)162301



Arleo,
EPJ C30(02)213

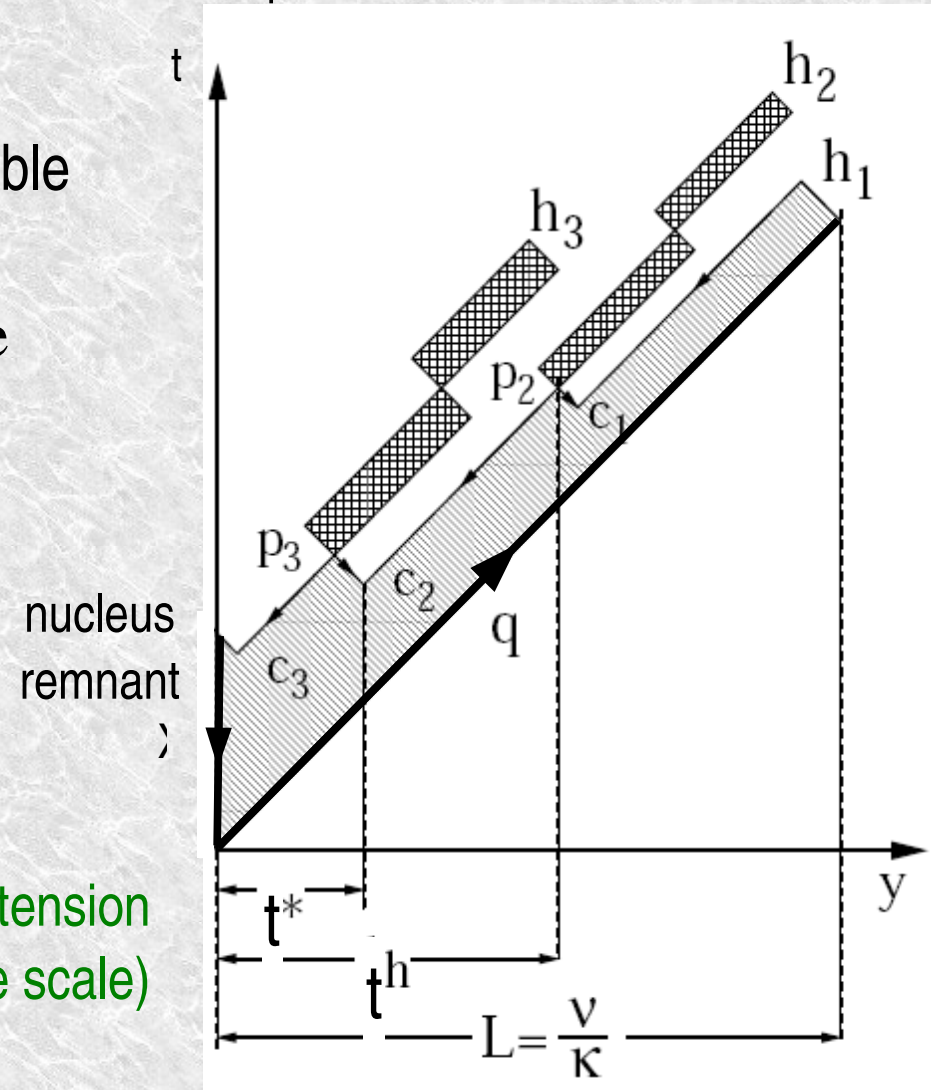
Formation time estimates 2 – Lund model

- Prehadrons and hadrons are identified as follows [Bialas-Gyulassy '87]
  - Prehadron formed at $q\bar{q}$ creation (string breaking) – C_i
  - Hadron h_i formed when q and q meet – P_i
- Average formation times analytically computable
  - <u>At large $z \to 1$</u>
    $E_h \to \nu \Rightarrow$ string breaks early to leave all energy to the hadron: $t^* \to 0$
  - <u>At small $z \to 0$</u>
    hadron created at high rank after many string breakings: $t^* \to 0$

$$\begin{cases} \langle t^* \rangle = f(z)\,(1-z)\,\dfrac{z\nu}{\kappa} \\ \langle t^h \rangle = \langle t^* \rangle + \dfrac{z\nu}{\kappa} \end{cases}$$

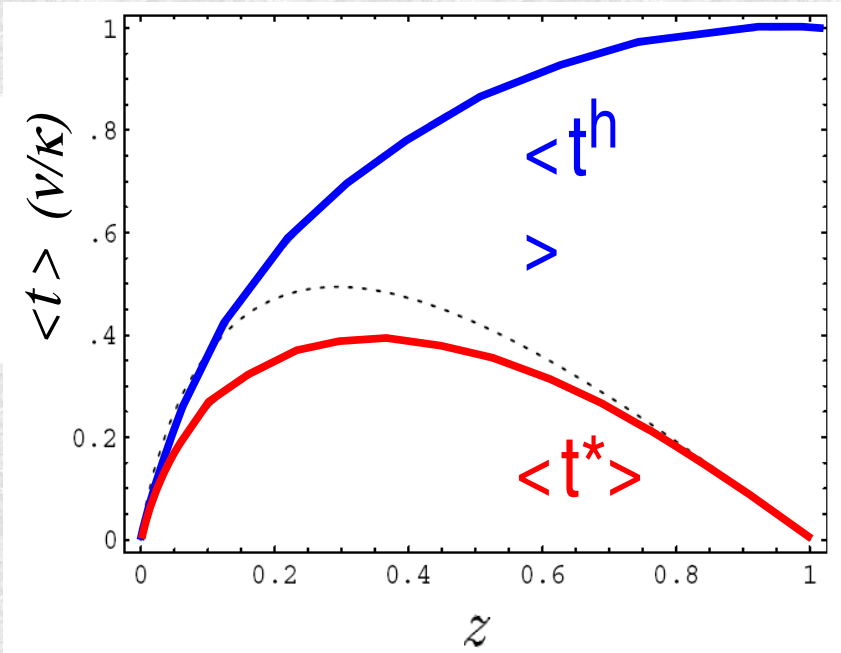
κ ← string-tension (non perturbative scale)

$(1-z)$ ← energy conservation



Formation time estimates 2 – Lund model

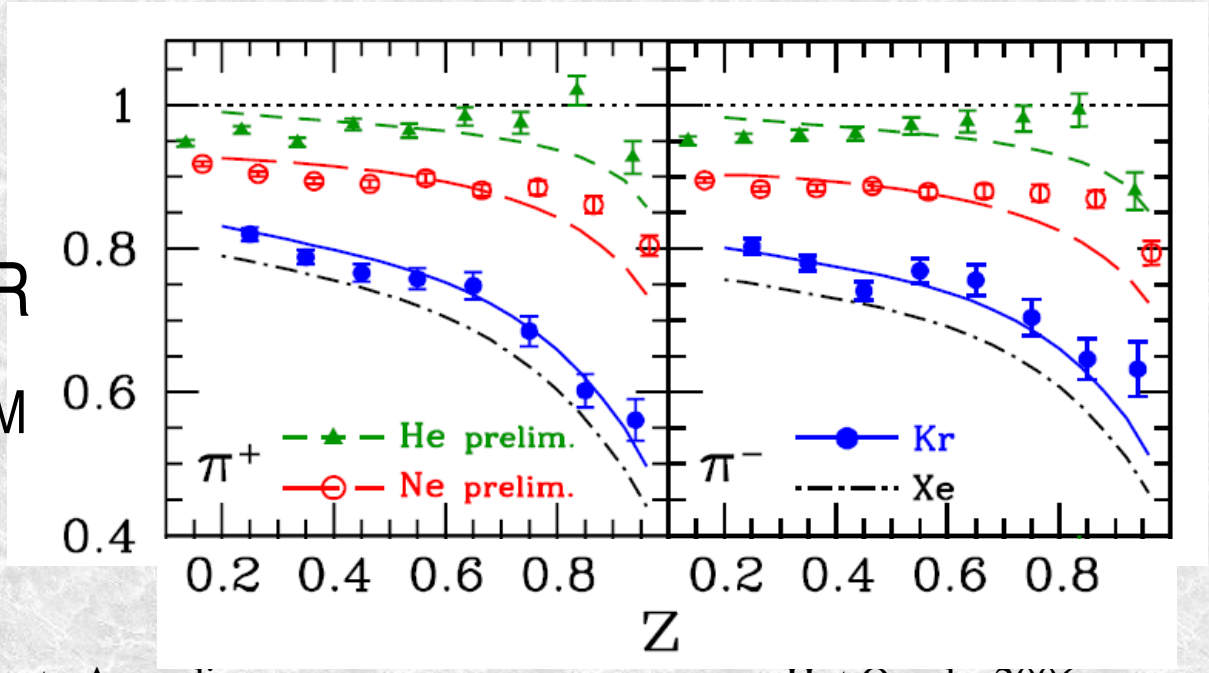
$$\begin{cases} \langle t^* \rangle = f(z)\,(1-z) \\ \langle t^h \rangle = t^* + \dfrac{z\nu}{1} \end{cases}$$



- For a 10 GeV pion at Hermes

  $t^* < 4$ fm ~ $O(R_A)$ $\quad t^h = 6\text{-}10$ fm $> R_A$

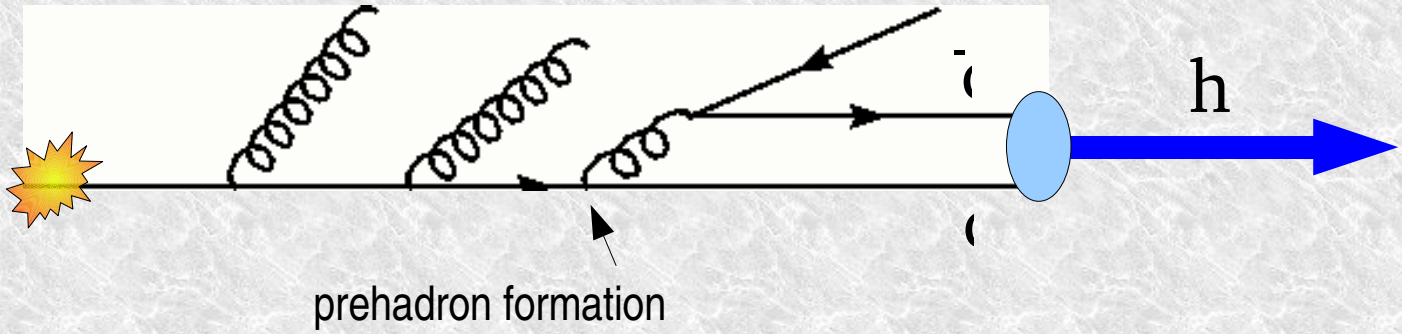
- Prehadron absorption with this estimate [A.A. et al., NPA 761(05)67]



see also:
Falter, Gallmeister, nucl-th/0512104
for similar ideas in a transport model

Formation time estimates 3 – Dipole model

- Leading hadron formation (z > 0.5) [Kopeliovich et al., NPA 740(04)211]



- Prehadron formation time t^* = time necessary to radiate $\Delta E = \nu - E_{qq}$
  - At large $z \to 1$
    $E_h \to \nu \Rightarrow$ quark must be short-lived
    (it can radiate only a few soft gluons)

$$\langle t^* \rangle \propto (1-z)\frac{z\nu}{Q^2}$$

energy conservation

boost

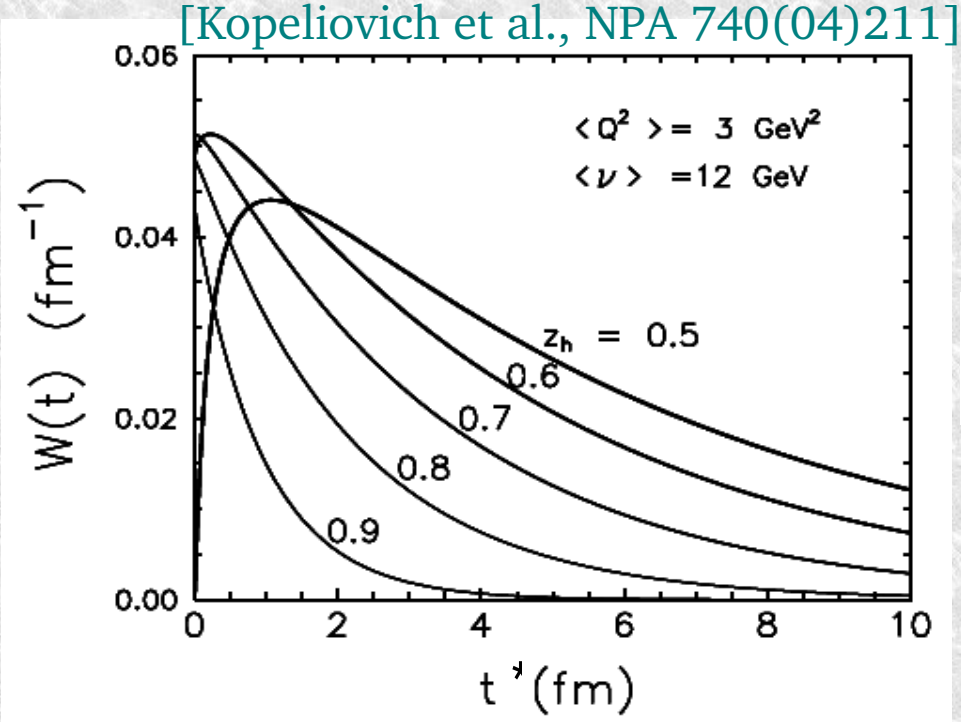
virtuality (perturbative scale)

- Evolution into hadron by path-integral formalism
  - usually $\langle t^h \rangle \gg R_A$

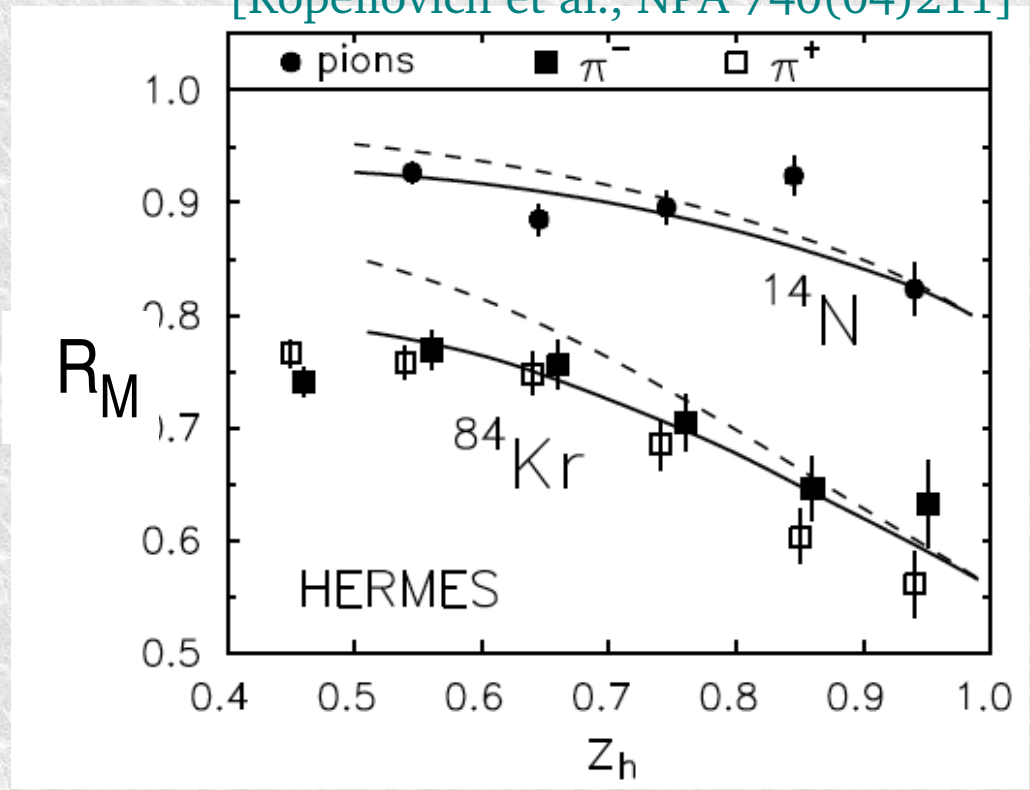
Formation time estimates 3 – Dipole model

probabilty distribution in t*

[Kopeliovich et al., NPA 740(04)211]



[Kopeliovich et al., NPA 740(04)211]



$$\langle t^* \rangle \propto (1-z)\frac{z\nu}{Q^2} < 5 \text{ fm} \quad (\text{at } z<0.5)$$

Note: computation of R_M includes: 1) prehadron absorption (dominant effect)

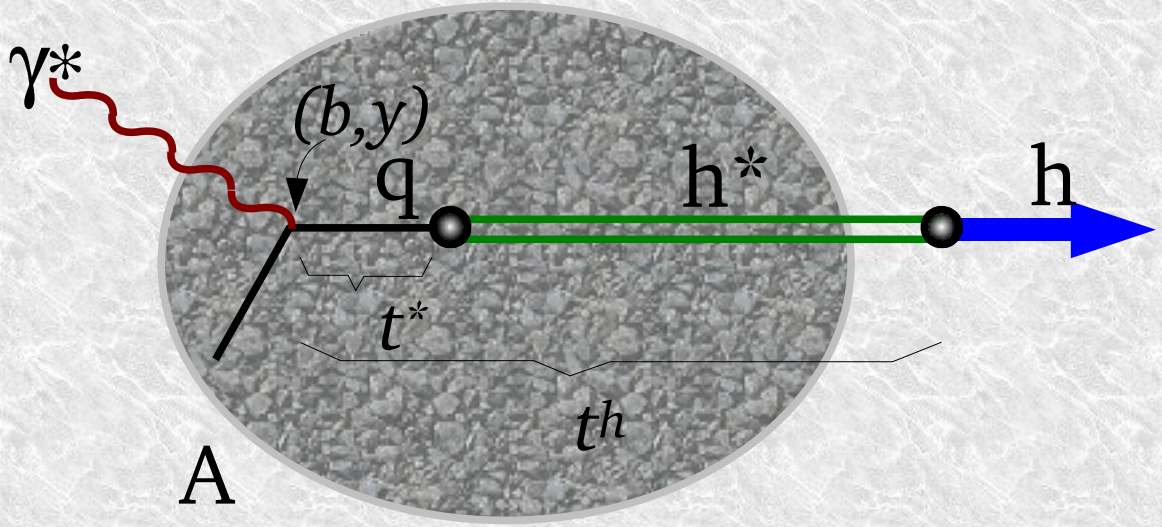
2) quark energy loss (subdominant)

III. Can we distinguish energy loss from prehadron absorption?

I - Hadron absorption model

Hadron absorption model

A.A. et al., NPA 761(05)67

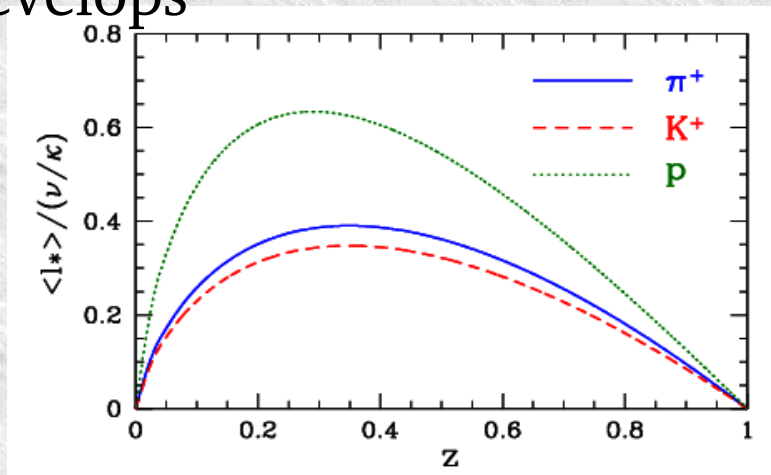


- Two-step hadronization inside the nucleus:
  1) quark q neutralizes color ⇒ **prehadron** h^*
  2) **hadron** h's wavefunction fully develops

- Average formation lengths

  $\langle t^* \rangle(z,\nu)$, $\langle t^h \rangle(z,\nu)$ from Lund model

"production length" "formation length"



Hadron absorption model - 2

A.A. et al., NPA 761(05)67

- (Pre)hadron suppression factor S_A by transport diff. eqns.
- (Pre)hadron-nucleon cross sections:

  $\sigma_* = 0.35\ \sigma_h$ - fitted to $e^+ + \mathrm{Kr} \to \pi^+ + \mathrm{X}$

  σ_h - from Particle Data Group
- Full integration over $\gamma^* q$ interaction point (b,y)

$$S^A_{f,h} = \int d^2b\, dy\, \rho_A(b,y) \int_y^\infty dx' \int_y^{x'} dx \frac{e^{-\frac{x-y}{\langle l^* \rangle}}}{\langle l^* \rangle} e^{-\sigma_* \int_x^{x'} ds A\rho_A(b,s)} \frac{e^{-\frac{x'-x}{\langle \Delta l \rangle}}}{\langle \Delta l \rangle} e^{-\sigma_h \int_{x'}^{\infty} ds A\rho_A(b,s)}$$

prob. that h^* is formed at x

absorption of h^* up to x'

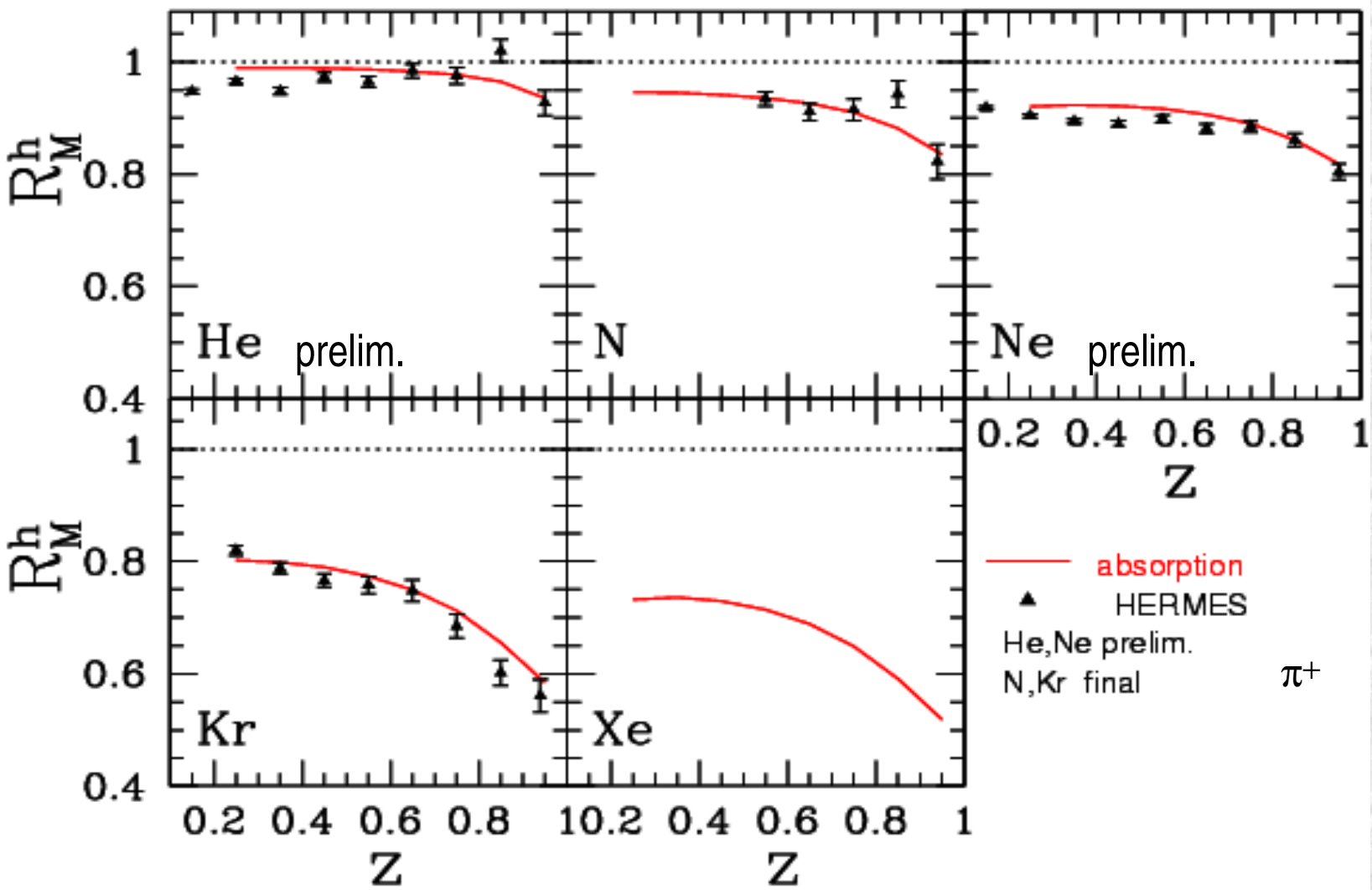
prob. that h is formed at x'

absorption of h from x' to ∞

$$\frac{1}{N_A^{DIS}} \frac{dN_A^h(z)}{dz} = \frac{1}{\sigma^{lA}} \int_{\text{exp. cuts}} dx\, d\nu \sum_f e_f^2 q_f(x,Q^2) \frac{d\sigma^{lq}}{dx d\nu} S^A_{f,h}(z,\nu) D^h_f(z,Q^2)$$

Hadron absorption model - results

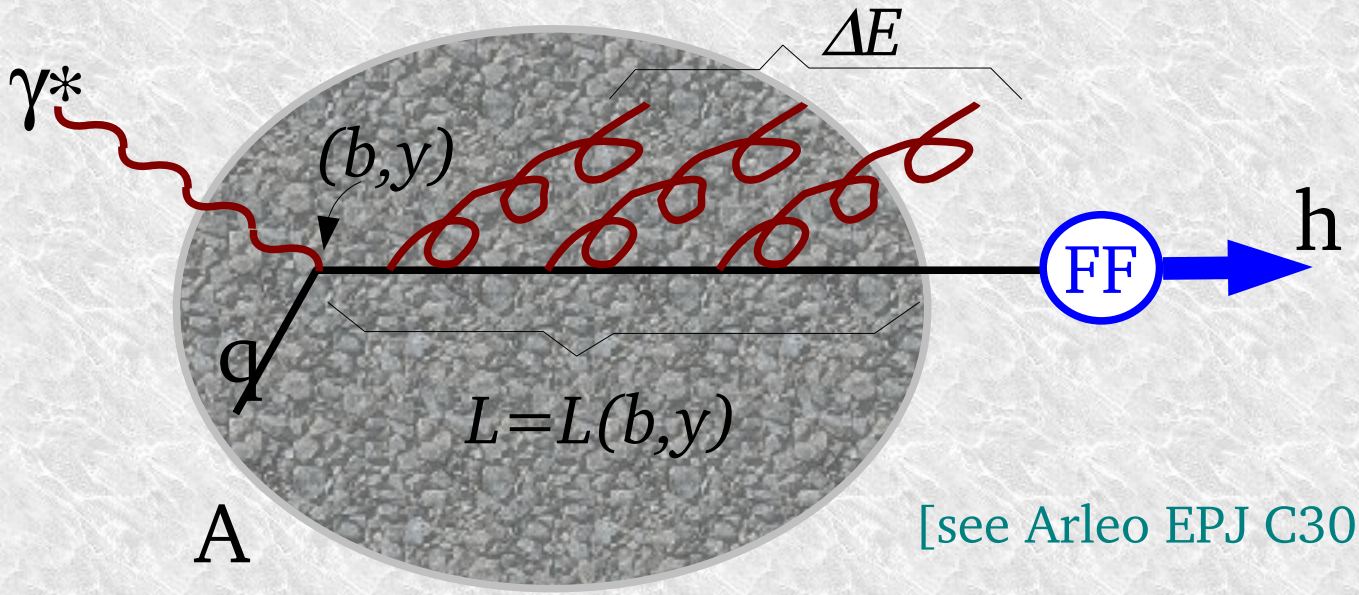
A.A. et al., NPA 761(05)67



Note: in ref. above, curves differ by inclusion of Q^2-rescaling due to additional hypothesis of partial deconfinement in nuclei

II - Energy loss model

Energy loss model



[see Arleo EPJ C30(02)213]

- The quark hadronizes outside the nucleus
- Gluon bremsstrahlung ⇒ ΔE ⇒ modified fragment. funct.

$$D_q^h(z,Q^2) \longrightarrow \frac{1}{1-\Delta z}\, D_q^h\left(\frac{z}{1-\Delta z}, Q^2\right) \quad ; \quad \Delta z = \Delta E/\nu$$

- *New*: use quenching weights $P(\Delta z, L)$ with corrections for finite in-medium path $L=L(b,y)$ [Salgado-Wiedemann,PRD68(03)162301]

$$\tilde{D}_f^h(z,Q^2;L) = \int_0^{(1-z)} d\Delta z\, \mathcal{P}(\Delta z;\hat{q},L)\frac{1}{1-\Delta z} D_f^h\left(\frac{z}{1-\Delta z},Q^2\right) + p_0(\hat{q},L)\, D_f^h(z,Q^2)$$

prob. of radiating Δz

prob. of radiating no gluons

Energy loss model - realistic geometry

- *New:* Full integration over $\gamma^* q$ interaction point (b,y)

$$\frac{1}{N_A^{DIS}}\frac{dN_A^h(z)}{dz} = \frac{1}{\sigma^{lA}}\int d^2b\, dy\, \rho_A(b,y)\int_{\text{exp. cuts}} dx\, d\nu \sum_f e_f^2 q_f(x,Q^2)\frac{d\sigma^{lq}}{dx d\nu}\tilde{D}_f^h(z,Q^2;L(b,y))$$

- Realistic nuclear density: Woods-Saxon parametrization for A>2

  Reid's soft-core for 2D

$$L(b,y) = 2\int_y^\infty dz\,(z-y)\rho(b,z) \Big/ \int_y^\infty dz\,\rho(b,z) \quad \left(= R(b) - y \ \text{ for Hard-Sphere}\right)$$

$$\langle\rho\rangle(b,y) = \int_y^\infty dz\,\rho(b,z) \Big/ L(b,y) \quad \left(= \rho_{HS} \ \text{ for Hard-Sphere}\right)$$

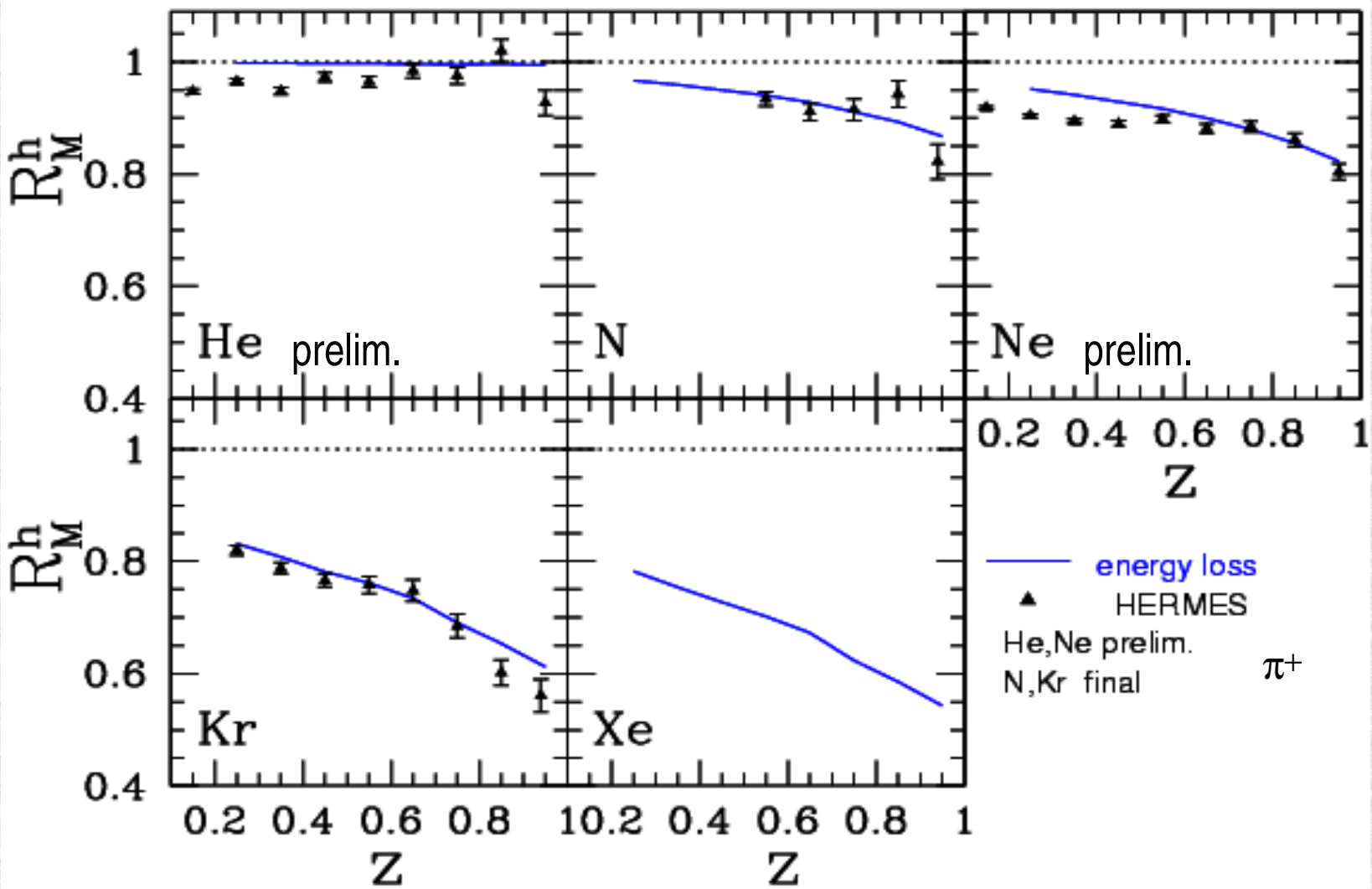
- Transport coefficient

$$\langle\hat{q}\rangle(b,y) = \hat{q}_0 \frac{\langle\rho\rangle(b,y)}{\rho(0,0)}$$

where $\hat{q}_0 = \langle\hat{q}\rangle(0,0)$

with $\hat{q}_0 = 0.5\ \text{GeV}^2/\text{fm}$ - fitted to $e^+ + \text{Kr} \to \pi^+ + X$

Energy loss model - results



III - The "$A^{2/3}$ power law"

Accardi, Gruenewald, Muccifora, Pirner, NPA 761(2005)67

A-dependence - naïve argument

At first order – i.e., for light nuclei:

a) Energy loss (LPM effect):

$$1-R_M \sim \langle \Delta z \rangle \sim L^2 \sim A^{2/3}$$

very naive,
incorrect,
but not too wrong...

b) Hadron absorption:

$$1-R_M \sim \langle \textit{no. of rescatterings} \rangle \sim L \sim A^{1/3}$$

WRONG!

$\Rightarrow$ a simple fit of $1-R_M$ to A^α should discriminate the 2 models

Let's really expand in powers of $A^{1/3}$

- Approximations for analytic formulae:
  - hard-sphere nuclei ($R_A = r_0 A^{1/3}$)
  - neglect nuclear effects on ^{2}H

- **Energy loss model**
  - neglect finite size corrections
  - large ν ⇒ neglect boundary in $\int_0^{1-z} d\Delta z$ - no energy conservation!

$$1 - R_M^{\text{en.loss}} = \underbrace{\frac{C_F \alpha_s r_0^2}{5} \frac{\hat{q}}{\nu} \left[-1 - z \frac{\partial_z D(z)}{D(z)} \right]}_{\substack{\text{coefficient is } z\text{-dependent} \\ \Rightarrow \text{ fragmentation dynamics}}} A^{2/3} + \text{h.o.t.}$$

Energy loss yields $A^{2/3}$ as expected at leading order

where do h.o.t. begin to break $A^{2/3}$?

Let's really expand in powers of $A^{1/3}$

- **Hadron absorption model**
  - prehadron formed inside A, hadron outside (it's a good approximation, see A.A. et al. NPA720(03)13)

$$1 - R_M^{\text{abs.}} = \underbrace{\frac{2\rho_0 r_0^2}{5} \frac{\sigma_*}{\langle l^* \rangle(z)}}_{\text{fragmentation dynamics}} A^{2/3} + \text{h.o.t.}$$

!!

Hadron absorption follows $A^{2/3}$ law, as well!

- to distuinguish energy loss and absorption:
  1) **check breaking of $A^{2/3}$ law**
  2) **don't forget the coefficient: it contains dynamics**

Why $A^{2/3}$ also for absorption?

- Absorption can, quite generally, be approximated in terms of

$$1 - R_M \approx \frac{\pi \rho_0}{A} \int_0^{R^2} db^2 \int_{-R(b)}^{R(b)} dy \int_y^{\infty} dx \, \underbrace{\mathcal{P}_*(x-y)}_{\text{prob. distrib. for h* production length}} \left[1 - e^{-\rho_0 \sigma_* \int_x^{\infty} ds \Theta(R(b) - |s|)} \right]$$

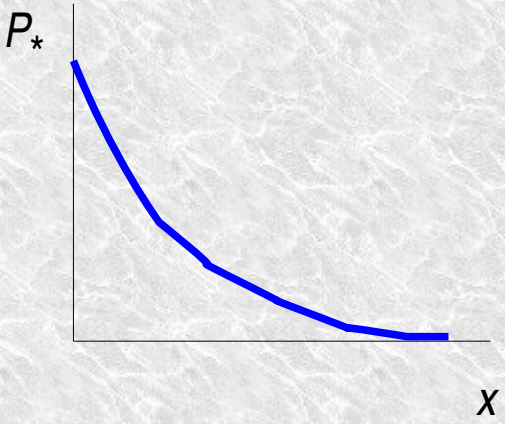
- If $\mathcal{P}_*(x-y) = \delta(x-y)$ $\Rightarrow 1 - R_M = c\, A^{1/3}$ (e.g. Falter's et al. leading h_*)
- If not, we have a dimensionful scale $\langle l_* \rangle = \int_0^{\infty} dx\, x\, \mathcal{P}_*(x)$
- After all integrations we obtain an extra power of A: $\left(\frac{R_A}{\langle l_* \rangle} \right)^n \propto A^{n/3}$
- Theorem: if $\mathcal{P}_*$ is normalizable $\Rightarrow$ $n>0$ (all known models n>1)

Why $A^{2/3}$ also for absorption?

- **Special cases**

1) $$\lim_{x\to 0} \mathcal{P}_*(x) = k \neq 0 \implies 1 - R_M = c\,A^{2/3} + \text{h.o.t.}$$

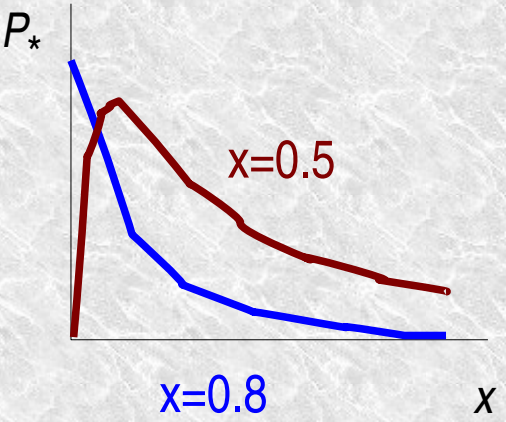
[e.g., the absorption model
A.A. et al. NPA '05]



2) $$\lim_{x\to 0} \mathcal{P}_*(x) = 0 \implies 1 - R_M = c\,A^{\alpha} + \text{h.o.t.}$$

[e.g., Kopeliovich et al. NPA...]

Not too different from case 1)



IV - cA^{α} fits

Accardi, Gruenewald, Muccifora, Pirner, NPA 761(2005)67

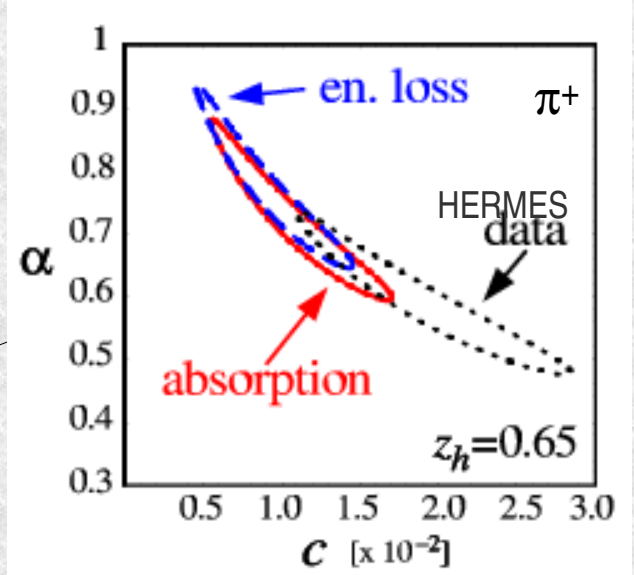
cA^{α} fits

- to distuinguish energy loss and absorption:
  1) **check breaking of $A^{2/3}$ law**
  2) **don't forget the coefficient: it contains dynamics**

- the simplest option:
  i) choose a set of nuclei $\{A_1, A_1, \ldots, A_N\}$
  ii) fit $1\text{-}R_M(z) = c(z)\, A^{\alpha(z)}$ as a function of A
  - at fixed z (or ν or Q^2)
  - with c and α as free parameters

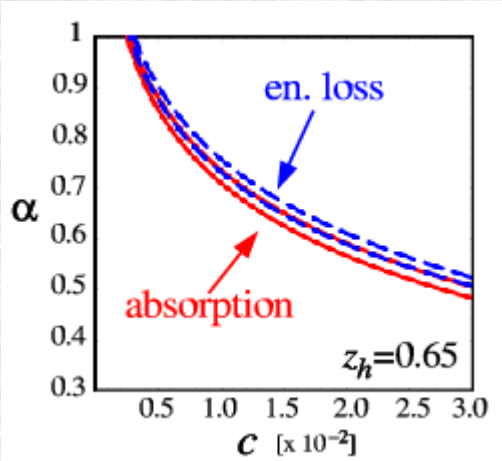
  iii) draw a 2σ confidence contour in the (c,α) plane

- But... still we cannot distinguish energy loss from absorption...
  [Accardi, QM2005, Acta Phys. Hung., in press]



So, can we distinguish...? Answer 1

- **Absorption and struck quark en.loss mimick each other**
  - not possible to distinguish (separate) the 2 mechanisms
  - not even using heavy nuclei up to Pb



A-dependence of R_M cannot be used
to test dominance of partonic or prehadronic physics